Substitution and income effects at the aggregate level:
The effective budget constraint of the government and the flypaper effect

Cristian F. Sepulveda *

Tulane University

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Abstract

This paper analyzes the effect of a change in lump-sum (private) income on the tax and expenditure decisions of a government constrained by taxpayers' behavioral responses to the tax policy. Lump-sum changes of private income are generally characterized as pure income effects on taxpayers' behavior, and usually considered in public finance theory as imposing no price effects on the economy. In this paper we show that pure income effects at the individual level can lead to three distinguishable effects at the aggregate level. The reason is that a change in lump-sum income affects taxpayers' behavioral responses to taxation and the size of the tax bases, altering the marginal cost of tax collections. The optimal fiscal responses of a welfare maximizing government can be broken up into a "net substitution effect," associated with a change in the marginal cost of public funds, a "private income effect," associated with the increase in private consumption, and a "public income effect," which is equivalent to the effect of intergovernmental transfers. As a consequence, the effects of lump-sum income and intergovernmental transfers on fiscal decisions are shown to be generally different, but consistent with empirical findings of the literature on the flypaper effect.

Keywords: flypaper effect, income and substitution effects, marginal cost of public funds, intergovernmental transfers

JEL codes: D78, H71, H77

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1. Introduction

Traditional microeconomic theory describes the individual demand response to a price change as composed by the income and substitution effects. These two effects have become so fundamental to basic economic analysis that they are often applied without regard to whether an economic decision is made by an individual or by a government. In this paper we show that the two traditional effects are in general insufficient to explain the behavior of a welfare maximizing government that faces aggregate responses to the fiscal policy. We focus on the effects of lump-sum (private) income on optimal fiscal decisions about the level of taxes and the amount of public expenditures. We show that even though changes of lump-sum income lead to very simple income effects on individual behavior, their impact at the aggregate level on the optimal government decisions is far more complex. This is because they affect taxpayers' behavioral responses to the tax policy, which in turn can alter the marginal cost at which the government finance any given level of public expenditures.

As explicitly recognized by traditional models of individual behavior, lump-sum income can affect individual decisions about consumption, time allocation and tax compliance. However, the literature does not explicitly acknowledge that as long as these behavioral responses alter the marginal cost of tax revenues faced by the government, then lump-sum income will impose a substitution effect on fiscal decisions. Moreover, whenever the marginal cost of tax revenues (in terms of private consumption foregone) is different from unity, the ability to purchase private goods and publicly provided goods will no longer vary in the same magnitude. At the aggregate level, therefore, in addition to the substitution effect, lump-sum income may also impose two different income effects that have also been overlooked in the literature.

The effect of lump-sum income on the marginal cost of tax revenues is an important but largely unexplored aspect of the positive problem of determining actual tax and expenditure policies. Changes in lump-sum income are generally regarded in public economic theory as imposing pure income effects, and for that reason they become uninteresting in a predominantly normative literature focused on minimizing distortions associated with substitution effects of price changes. Samuelson's (1954) condition describes the first best solution to the problem of determining the optimal amount of public goods and tax levels under (supposedly efficient) lump-sum taxation, a condition that is valid after any redistribution of lump-sum income. On the other hand, the second best solution to the problem, analyzed in the marginal cost of public funds literature, deals with the more realistic case where lump-sum taxation is unavailable and the government depends on taxes that impose substitution effects on taxpayers' behavior, such as those on labor income or on consumption (Atkinson and Stern 1974; Ballard and Fullerton 1992; Auerbach and Hines 2002; Dahlby 2008).

In this paper we analyze the optimal behavior of a welfare maximizing government from a positive perspective. Our main concern is the sign and relative magnitude of the effects of lump-sum income on optimal tax and expenditure decisions, provided that a government is constrained in its ability to raise tax revenues. The paper contributes to the literature by identifying three distinguishable effects of lump-sum private income on the optimal fiscal decisions of the government. We show that one of these three effects is equivalent to the effect of direct transfers to the government, which implies that a change in taxpayers' lump-sum income and direct transfers to the government generally have very different effects on optimal government decisions.

The literature on the flypaper effect has extensively studied the effects of private income and intergovernmental transfers on fiscal decisions of state and local governments. The flypaper effect is an empirical regularity whereby a given amount of intergovernmental transfers received by a government has a greater impact on public expenditures than an identical increase of private income within the same jurisdiction. [1] This literature provides strong evidence against the "veil hypothesis," also called the "equivalence theorem," according to which the effect of a given amount of income on fiscal decisions should be the same regardless of whether the money is received by the government or by the individuals (Bradford and Oates 1971). Under that hypothesis transfers are considered to be equivalent to (or a "veil" for) an increase of private income.

During the last four decades there have been many attempts to explain the flypaper effect theoretically (see Hines and Thaler 1995; Bailey and Connolly 1998; Gamkhar and Shah 2007), but the literature has not yet reached a definite consensus. Specially for this paper is the work of Dahlby (2011), who builds upon the work of Hamilton (1986) to argue that lump-sum transfers can stimulate public expenditures more than private income increases because they normally lead to a greater reduction of the marginal cost of public funds. A similar approach is followed by

[1] A revision of the early empirical literature can be found in Gramlich (1977); more recent empirical studies are surveyed in Gamkhar and Shah (2007).

Aragon (2012), who argues that the flypaper effect can be fully explained if the marginal cost of tax collections increases with the tax rate. This explanations challenge one of the most accepted theories explaining the flypaper effect, based on the "fiscal illusion hypothesis." Fiscal illusion models presume that the median voter or the representative taxpayer perceives only the average cost of public goods and services and, as a result, he underestimates their real marginal cost and mistakenly chooses to overspend (Oates 1979, Logan 1986, Turnbull 1998). In contrast, Dahlby's (2011) explanation requires no misperception from local agents, who correctly choose to spend a high proportion of transfers on public goods and services because their marginal cost has in fact been reduced by the transfers.

This work extends the analysis of Hamilton (1986) and Dahlby (2011), who focus on the marginal welfare cost of public expenditures to explain the flypaper effect. We use a very simple welfare maximization framework, where the tax base changes with the taxpayers' behavioral responses to the tax rate. Assuming identical taxpayers and a linear production technology we describe, formally and diagrammatically, the three effects of lump-sum income on the optimal fiscal policy. One effect is labeled "private income effect," and represents the change in the demand for public goods associated with a greater purchasing power for private goods. The "public income effect" represents the adjustment in demand due to a change in the purchasing power for public goods, and it is equivalent to the effect of intergovernmental transfers of a given amount. This effect will be present whenever the initial change in lump-sum (private) income has an impact on the size of the tax base. Finally, the "substitution effect" of lump-sum income responds to the possible change induced on marginal tax revenues. The amount of revenues collected with marginal increases of the tax rate implicitly defines the marginal cost of public funds; if that amount changes with the level of taxation, so will the marginal cost of public funds, and lump-sum income will have a substitution effect on optimal fiscal policy.

The remainder of the paper is organized as follows. The next section develops the theoretical model and derives the three effects of lump-sum income on government decisions. Section 3 presents the diagrammatic analysis, where we introduce the effective budget constraint and describe sufficient conditions to observe the flypaper effect. Section 4 concludes.

2. Fundamental substitution and income effects of lump-sum income at the aggregate level

This section presents the theoretical model and derives the main results of the paper, which are the substitution and income effects of lump-sum (private) income on the optimal level of public expenditures. The government is assumed to maximize a utilitarian welfare function Ω describing the preferences for private goods x and public expenditures G of N identical taxpayers.[2] This framework is similar to the framework used in the flypaper literature, where relevant social preferences are typically reduced to those of the median voter or the representative taxpayer, while leisure is excluded from the utility function to simplify the analysis.

The representative taxpayer receives two types of income. One varies with behavioral responses to the tax and expenditure policies and is taxable; so it corresponds to the individual tax base b. The other is an exogenous and untaxed lump-sum amount denoted by z. Assuming no savings the individual consumption of private goods is $x = (1-t)b + z$. The government collects own tax revenues by applying a tax rate t on the available tax base $B = Nb$, and may also receive intergovernmental transfers S. If in addition we assume that the marginal rate of transformation between private and public goods and services is constant and equal to one (linear and unitary production technology), then the total amount public goods' provision G is equal to the amount of tax revenues $R = tB + S$, which for any given S is fully determined by the choice of t. Assuming also for simplicity that taxpayers' preferences are additively separable, the government problem and the first order condition can be expressed as:

$$\max_{t} \ \Omega = N(u^x\{(1-t)b + z\} + u^G\{tB + S\}) \ , \tag{1}$$

$$N[-b + (1-t)b_t^*]u_x + (B + tB_t^*)Nu_G = 0 \ , \tag{2}$$

where subscripts represent partial derivatives and asterisks represent optimal decisions, which in this case correspond to the taxpayers' behavioral responses to the tax policy t. We assume that the amount of intergovernmental transfers S does not directly affect taxpayers' behavior,[3] and use the implicit function theorem to compute the marginal effect of S on the optimal tax rate t^*:

[2] With few exceptions, we will generally use lowercases to represent individual variables and capital letters to represent aggregate variables.

[3] There are several ways in which transfers can affect taxpayers' behavior. Transfers can be spent on goods and services that influence economic activity, private income, or even the taxpayers' perception of government performance and their willingness to pay taxes; or alternatively, they can be used to improve the administration,

$$\frac{dt^*}{dS} = \frac{R_t^* N u_{GG}}{(-d^2\Omega/dt^2)}\,, \tag{3}$$

where $R_t^* = B + tB_t^*$ and the denominator must be positive subject to the second order condition for welfare maximization. If $R_t^* > 0$, as we should expect since the very objective of the tax is to collect more revenues, and under diminishing utility from public goods and services ($u_{GG} < 0$), the optimal tax rate t^* decreases with intergovernmental transfers S. Even though this result does not necessarily hold when preferences are not separable, it does show how intergovernmental transfers can crowd out own tax revenues. Given that tax collections are costly in terms private goods forgone, the transfers received (which presumably have little or no costs for the local community) will allow to increase the consumption of both private and publicly provided goods.

Applying again the implicit function theorem to (2) and using (3), the marginal effect of increasing z for the N taxpayers on the optimal tax rate t^* is: [4]

$$N\frac{dt^*}{dz} = \frac{\left[R_{tz}^*\left(N\frac{u_G}{u_x}-1\right)+B_{tz}^*\right]u_x}{(-d^2\Omega/dt^2)} + \frac{X_z^* X_t^* u_{xx}}{(-d^2\Omega/dt^2)} + R_z^*\frac{dt^*}{dS}\,. \tag{4}$$

This expression describes the effect of Nz on t^*, which consists of three separable effects. The first term on the right hand side represents a net substitution effect, explained by the change of the relative price of public expenditures in terms of private goods forgone. This effect would be negative at the optimal solution if $Nu_G/u_x > 1$, which is the likely the case in a second best scenario where government expenditures are not tax productive and taxation is costly, and z has a negative effect on marginal tax revenues ($R_{tz}^* < 0$) and the marginal tax base ($B_{tz}^* < 0$). Note that the substitution effect could also be positive, in which case the optimal tax rate would increase with z.

In general, if the government is unable to transform one monetary unit of private income into exactly one monetary unit of public expenditures, then an additional amount of lump-sum income will have a different impact on the purchasing power for private and publicly provided goods, leading to two distinguishable income effects. The second term on the right hand side of (4) can be defined as a "private income effect," which captures the influence of a greater purchasing power for private goods on t^*. Plausible conditions are given by $X_z^* > 0$, $X_t^* < 0$ and

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collection and enforcement of current taxes. In addition, the transfers themselves may have been collected via taxation and thus associated with their own behavioral responses to the tax policy and marginal tax collection costs. However, these costs are faced by the government unit providing the transfer in a previous stage of the budgeting process, and so they can be considered as irrelevant by the recipient government.

[4] A detailed derivation is available in Appendix I.

$u_{xx} < 0$, in which case additional lump-sum income would increase t^*, such that part of the additional taxpayers' wealth is used to finance an increase of public expenditures. Finally, the last term of (4) is a "public income effect," which represents the effect of an increase of purchasing power for public expenditures goods on t^*. In fact, this effect is equivalent to the effect of transfers given directly to the government, defined in (3), multiplied by an amount equal to the marginal change of revenues due to the lump-sum income increase, R_z^*.

The optimal tax policy t^* is associated with one unique optimal amount of public expenditures G^*, which is therefore determined by the same substitution and income effects. Using the balanced budget condition, $G = tB + S$, the responses of optimal public expenditures to equivalent changes of S and Nz are given by: [5]

$$\frac{dG^*}{dS} = \frac{dt^*}{dS} R_t^* + 1 \,, \tag{5}$$

$$N \frac{dG^*}{dz} = \frac{\left[R_{tz}^*\left(N\frac{u_G}{u_x} - 1\right) + B_{tz}^*\right]u_x}{(-d^2\Omega/dt^2)} R_t^* + \frac{X_z^* X_t^* u_{xx}}{(-d^2\Omega/dt^2)} R_t^* + R_z^* \frac{dG^*}{dS} \,. \tag{6}$$

Equation (5) shows that the marginal effect of S on G^* consists of two components. One is described by the number 1, and represents the fact that, under a linear and unitary production technology, one monetary unit received in the form of intergovernmental transfers can be transformed into one monetary unit of public expenditures. This component is equivalent to a subsidy on the provision of public goods and services. The other component is given by the change in tax revenues due to the marginal impact of S on the optimal tax rate. Provided $dt^*/dS < 0$, intergovernmental transfers will most likely increase G^*, but in an amount smaller than the transfer itself. In other words, a share of the additional transfers will be used to finance private consumption.

Equation (6) describes the three effects of Nz on G^*. Provided $Nu_G/u_x > 1$, if Nz erodes the marginal tax collections ($R_{tz}^* < 0$) and the marginal tax base ($B_{tz}^* < 0$), then increasing Nz will make public expenditures more expensive at the margin, and G^* would be reduced by virtue of the substitution effect. The private income effect, defined by the second term on the right hand side, represents the effect of increasing the purchasing power for private goods. As private consumption increases, the demand for public expenditures will also increase if this term is positive, pushing G^* up. The public income effect of Nz on G^* is represented by the last term on

[5] Derivations available in Appendix II.

(6). The change of government revenues, due to the marginal increase of lump-sum income (R_z^*), will affect G^* as if the same amount of revenues was given directly to the government in the form of transfers S. Overall, the final magnitude and sign of the net effect of Nz on G^* is uncertain, and critically depends on how Nz affects taxpayers' behavior. The private income effect would be positive under our simplifying assumptions ($X_z^* > 0$, $X_t^* < 0$, $u_{xx} < 0$), but it may plausibly be counterbalanced by the substitution and public income effects if Nz erodes revenue collections and so either R_{tz}^*, B_{tz}^* or R_z^* are negative.

The identification of the three effects of lump-sum income on the optimal amount of public expenditures is important because it allows us to better understand government fiscal decisions, as well as the differences between economic choices at the individual and aggregate levels. This framework also helps to explain the flypaper effect, as it clearly describes the different channels through which lump-sum income and intergovernmental transfers can affect government behavior. Provided that intergovernmental transfers do not directly affect taxpayers' behavioral responses to taxation, equations (5) and (6) suggest that they have only a pure income effect on the amount of public expenditures. [6] This analysis helps to clarify the nature of the price effects of intergovernmental transfers previously described in the literature to explain the flypaper effect. The price effect described, for instance, by Oates (1979) and Dahlby (2011), are equivalent to a subsidy that reduces the marginal cost faced by the government at each level of public expenditures, but it is conceptually different from a substitution effect. It is true that the final marginal price of public expenditures is reduced with intergovernmental transfers (Dahlby 2011), and that this change can be represented by a movement along the demand function for public goods and services (Oates 1979), but this is not equivalent to the substitution effect described in (6), which does not directly depends on intergovernmental transfers S. In the next section we provide a diagrammatic explanation of this argument.

3. Diagrammatic analysis of optimal fiscal decisions and the flypaper effect

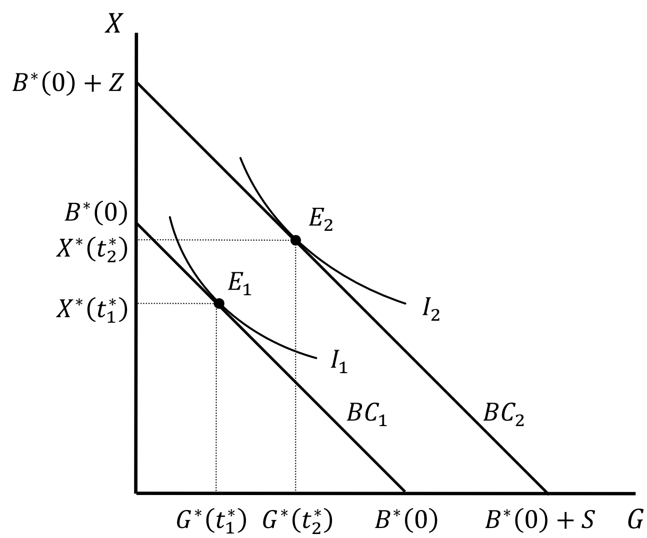
This section presents a diagrammatic analysis of the effects of lump-sum (private) income and intergovernmental transfers on optimal fiscal decisions. We introduce a graphical representation

[6] Recall that intergovernmental transfers in this model do have an indirect effect on taxpayers' behavior, through their effect on the optimal tax policy.

of the “effective” budget constraint of the government, and we use it to describe the substitution and income effects defined in the previous section, as well as to explain the flypaper effect.

In Figure 1 we represent the traditional approach to the theory of subnational government finances, which generally follows Bradford and Oates (1971) and is characterized by the fiscal equivalence of lump-sum income Z and intergovernmental transfers S. An implicit assumption of this approach is that the government is able to transform one monetary unit of private consumption X, in the vertical axis, into one monetary unit of public expenditures G, in the horizontal axis. The slope of the budget constrain of the government (BC), therefore, is constant and equal to minus one.

Figure 1: Optimal government decisions under costless tax collections
(traditional theory of subnational government finances)



The original equilibrium is found at E_1, where the social indifference curve I_1 is tangent to the budget constraint BC_1, which is here described for the case where $Z = S = 0$. This solution is associated with the optimal tax rate t_1^*, which simultaneously determines private consumption $X^*(t_1^*)$ and public expenditures $G^*(t_1^*)$. Given that tax collections are assumed to be costless, equal changes of Z and S increase the purchasing power in the private and public sectors equally, and both variables independently shift the budget constraint to BC_2. The new equilibrium will be at E_1, regardless whether additional income is received by individuals as Z or by the government as S, where private consumption and public expenditures have increased to

$X^*(t_2^*)$ and $G^*(t_2^*)$, respectively. The tax rate might decrease, increase, or remain at the same level, depending on whether the taxpayers value private consumption more, less, or the same as public expenditure.

The literature on the flypaper effect provides strong empirical evidence against the hypothesis of fiscal equivalence between lump-sum income Z and intergovernmental transfers S, as the latter is found to have a greater stimulative effect on public expenditures than the former. In this paper we argue that the traditional approach fails to explain the flypaper effect because it does not recognize that the marginal cost of public expenditures is endogenous to the government's fiscal decisions. In this simple model, the marginal cost of public expenditures is expressed in terms of the private consumption that is sacrificed to collect an additional unit of revenue, and corresponds to the slope of government's budget constraint in the private-public goods space. At the optimal solution the slope of the budget constraint must be equal to the slope of the of the highest attainable social indifference curve. Rearranging the first order condition of the government's problem in (2) we can represent this solution as:

$$N\frac{u_G}{u_x} = -\frac{X_t^*}{G_t^*} = \frac{B-(1-t)B_t^*}{B+tB_t^*}\,, \tag{7}$$

where the sum of the marginal rate of substitution between public and private goods (the slope of the highest attainable social indifference curve) is equal to the marginal cost of public expenditures in terms of private consumption (the slope of the government's budget constraint). Equation (7) describes a simplified version of the adjusted Samuelson's (1954) condition, a well known result from the (normative) literature on the marginal cost of public funds (MCF), although slightly modified to account exclusively for the (positive) behavioral constraints faced the government. [7] It is easy to verify that the last expression in (7) will take the value of unity when, as implicitly assumed under Bradford and Oates' (1971) veil hypothesis, the tax rate has no effect on taxpayers' behavior and the size of the tax base ($B_t^* = 0$).

Now we turn to analyze the optimal choice of the government when the tax rate does have an effect on taxpayers' behavior and the size of the tax base ($B_t^* \neq 0$). Here we define the

[7] The adjusted Samuelson condition defines the MCF, the right hand side expressions in (7), as the welfare cost of an additional unit of tax revenues. If $B_t^* < 0$ ($B_t^* > 0$), then the MCF as defined in (7) would be greater (smaller) than the traditional definition, which incorporates the welfare gains associated with a reduction of the tax base in the numerator. For instance, under labor income taxation, the first order condition of the time allocation problem states that $(1-t)wu_x = u_l$, where is the utility of labor; when leisure utility is considered in the analysis, this condition implies that the term $-(1-t)B_t^*$ in the numerator is cancelled out. For more details, see Auerbach and Hines (2002) and Dahlby (2008).

effective budget constraint of the government (EBC) as the line encompassing the set of affordable combinations of private consumption and public expenditures $[X^*(t), G^*(t)]$, after accounting for the tax base adjustments due to taxpayers' behavioral responses to the tax rate.

In Figure 2(a) we present a simple case in which the marginal cost of public funds (MCF), as defined by (7), is greater than one (implying $B_t^* < 0$) and constant, such that any additional monetary unit of public expenditures requires the government to sacrifice more than one monetary unit of private consumption. The original budget constraint is given by EBC_3, which starts in the vertical axis at point $[B^*(0), 0]$ with no taxes or public sector, and ends at point $[X^*(1), B^*(1)]$. [8] The optimal solution is $[X^*(t_3^*), G^*(t_3^*)]$, at E_3, where EBC_3 is tangent to I_3. Note that the solution described in Figure 1 by E_1 is unaffordable whenever the MCF is greater than one and constant.

Under costly tax collections, equal amounts of transfers given to the taxpayers (in the form of Z) or to the government (S) will have different effects on the purchasing power of private and publicly provided goods. In what follows we assume for simplicity that Z does not affect either the tax base or the behavioral responses to taxation, so that $B_z^* = B_{tz}^* = 0$ and thus $R_z^* = R_{tz}^* = 0$ as well. This assumption is convenient to separate the three effects of Z on the optimal public expenditure decisions. If Z has no effect on the behavioral responses to taxation then the substitution effect described in (6) is zero. Similarly, if Z has no effect on the tax base then the public income effect described in (6) is also zero. As a result, this assumption allows us to isolate the private income effect of Z on G^*.

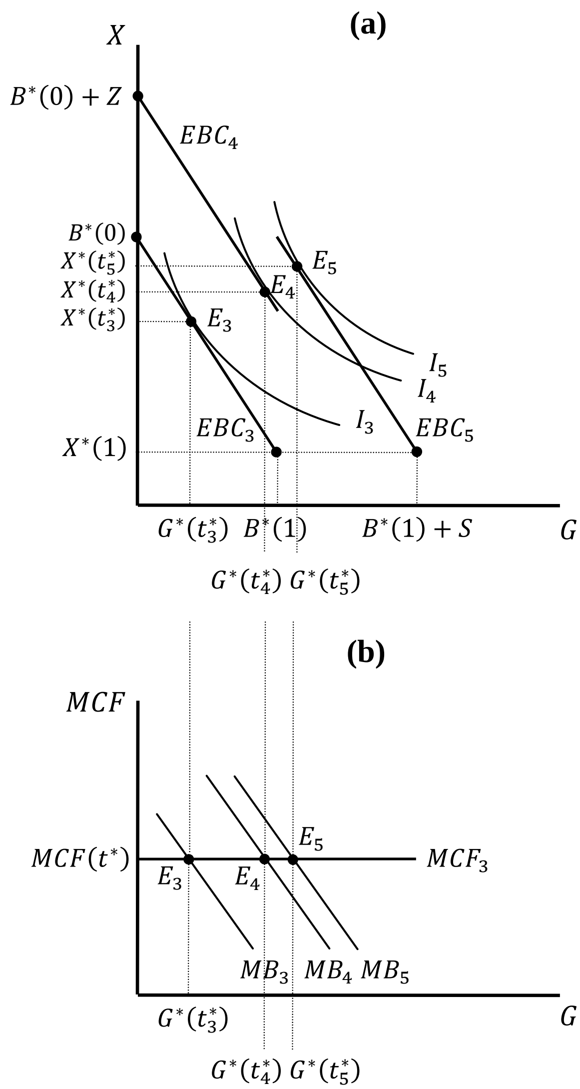
In this context, an amount of transfers Z given directly to the taxpayers can be represented as an upward shift of the effective budget constraint to EBC_4. The new solution is at E_4, associated in this case with tax rate t_4^*, which according to the private income effect in (4) is greater than t_3^* . [9] Similarly, the private income effect in (6) predicts that the new amount of public expenditures, $G^*(t_4^*)$, is greater than $G^*(t_3^*)$. Moreover, given that the MCF has remained constant, the increase of public expenditures can be fully explained by a demand increase. The demand shift is represented in Figure 2(b) by the distance between marginal benefit functions

[8] Private consumption can be positive under a tax rate of 100% if part of the original base is deviated towards untaxed activities.

[9] Graphically, the value of the tax rate corresponds to the vertical distance between the intercept of the effective budget constraint EBC and the optimal solution, divided by the full vertical height of the EBC, generally defined as $X^*(0) - X^*(1)$.

MB_3 and MB_4, while the marginal cost of funds' function MCF_3 represents a perfectly elastic supply curve that is invariable with respect to the position of the effective budget constrain.

Figure 2: Optimal tax and expenditure decisions under MCF > 1 and <u>*constant*</u>



If, instead, an amount of transfers $S = Z$ is given to the government, then the purchasing power of public goods and services would increase in the same amount of the transfer and the effective budget constraint would shift horizontally from EBC_3 to EBC_5. The new equilibrium would be found at E_5, where public expenditures have increased to $G^*(t_5^*)$ –more than with the rise under individual transfers Z; but the tax rate, now at t_5^*, has been reduced as predicted in (3). Again, as shown in Figure 2(b), the change in public expenditures can be fully attributed to the expansion in demand, which this time has been produced by the public income effect. Furthermore, the predicted effects of Z and S on the optimal tax and expenditure decisions are compatible with the results found in the flypaper effect literature, which are surveyed, for instance, by Gamkhar and Shah (2007). We can conclude that a MCF constant and greater than one is sufficient to produce that outcome. This simple condition expands the applicability of the model beyond the condition described by Dahlby (2011), who suggested that in order to generate a flypaper effect the MCF must be increasing in t.

For the following discussion we maintain the same simplifying assumptions regarding the nil effect of Z on taxpayers' behavior ($B_z^* = B_{tz}^* = 0$), but we allow for the MCF to vary with the level of the tax rate. In particular, like Dahlby (2011), we assume that the MCF increases with t. [10] A higher tax rate increases the return of various forms of tax avoidance, like labor supply reduction, doing-yourself activities, and mobility towards lower-taxed jurisdictions. As long as a higher tax rate increases tax avoidance, each additional unit of revenue will require forgoing more private goods. This case is of special interest since the incentives to engage in tax avoidance and untaxed activities can be expected to change with the tax rate. As pointed out by Slemrod and Kopczuk (2002) in the context of income taxation, the elasticity of the tax base is not immutable (and consequently neither the measure of the MCF) because it depends on behavioral responses that may be far from negligible.

Mathematically, if the MCF is increasing in t then the effective budget constraint is strictly concave, meaning that its (negative) slope m decreases with t: [11]

$$\frac{dm(t)}{dt} = (B^* B_{tt}^* - 2B_t^{*2})/R_t^{*2} < 0\,. \tag{8}$$

Regardless of the value of B_t^*, $B_{tt}^* \leq 0$ is a sufficient condition for the strict concavity of the effective budget constraint; or alternatively, if B_t^* is assumed to be constant ($B_{tt}^* = 0$), then any $B_t^* \neq 0$ leads to an strictly concave effective budget constraint. Graphically, EBC_6 in Figure 3(a) represents a strictly concave effective budget constraint, which roughly resembles a Laffer curve where (flipping the graph ninety degrees to the left) tax revenues are shown to increase at a decreasing rate as the tax rate increases in the axis measuring private consumption.

[10] This assumption is also very similar to the one used by Hamilton (1986), who analyzed a scenario where the marginal deadweight loss of taxation is an increasing function of the tax rate. In this paper we follow a more positive approach and focus instead on the effects of the tax rate on the behavioral determinants of the tax base.

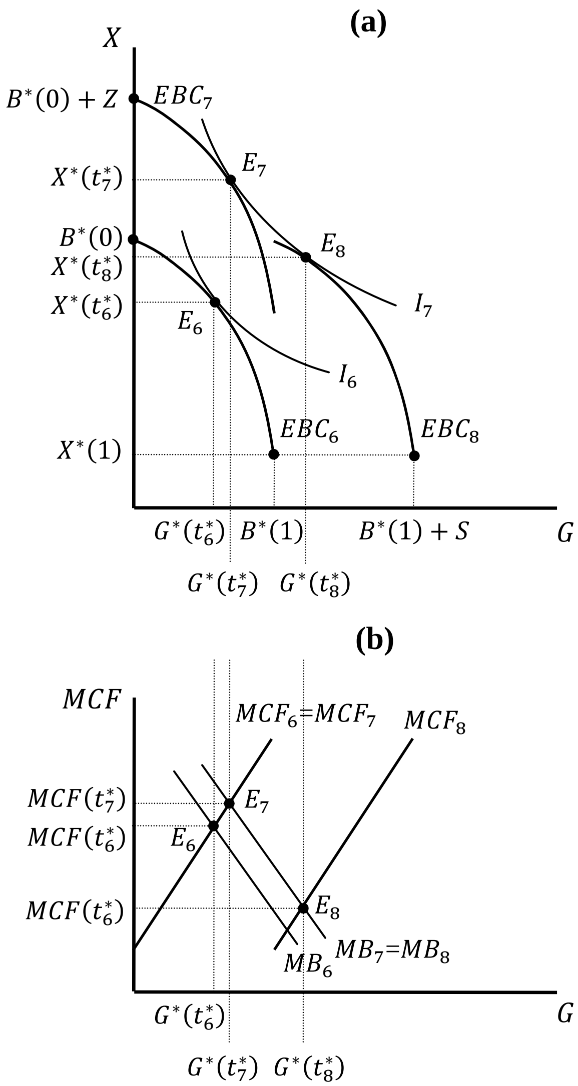
[11] See derivation in Appendix III.

The initial equilibrium is found at E_6, where EBC_6 is tangent to the highest attainable indifference curve I_6; at this point the optimal tax rate is t_4^* and the optimal combination of private and publicly provided goods is $[X^*(t_4^*), G^*(t_4^*)]$. An increase of transfers to the individuals by Z shifts the effective budget constraint vertically to EBC_7. This vertical shift represents a pure private income effect, which in this case increases the demand for public goods and services from MB_6 and MB_7 in Figure 3(b). As a result, the optimal tax rate, the MCF and optimal public expenditures, all increase up to t_7^*, $MCF(t_7^*)$ and $G^*(t_7^*)$, respectively. Note that the rise of the MCF attenuates the increase of optimal public expenditures, and that this "price effect" is not a substitution effect, but instead the result of stimulating the demand of public goods and services through the private income effect.

An equal increase of intergovernmental transfers S will shift horizontally the effective budget constraint from EBC_6 to EBC_8 in Figure 3(a), as well as the marginal cost of funds' function from MCF_6 to MCF_8 in Figure 3(b). The new equilibrium is represented by E_8, which for convenience is assumed to be tangent to indifference curve I_7 in Figure 3(a). Given that under our assumptions neither Z or S affect the tax base, and that they allow to reach the same level of welfare at I_7, then both Z and S lead to a shift of MB_6 to MB_7 in Figure 3(b). As long as S increases the demand for public goods; therefore, it will necessarily lead to an expansion of public expenditures G^*. On the other hand, the final effects on the tax rate and the MCF will depend on the relative size of the supply (MCF) and demand (MB) shifts and the slope of these functions, although in our simplified model –according to (3)– the tax rate and the MCF will be reduced as long as $u_{GG} < 0$.

The analysis so far has been restricted to the case in which lump-sum income has no effect on taxpayers' behavior and the tax base. In practice, however, lump-sum income will induce taxpayers to reevaluate their labor and tax compliance choices, affecting the size of the tax base and their behavioral responses to taxation. In that context, lump-sum income would affect not only the position of the MB function in Figure 3(b), but also the position of the MCF function due to the public income effect, and the slope of the two functions due to the substitution effect described in (6).

Figure 3: Optimal tax and expenditure decisions under <u>*increasing*</u> MCF



This analysis demonstrates that lump-sum income of individuals generally does not translate into a single lump-sum income effect at the aggregate level, as implicitly assumed in models based on a representative taxpayer or the median voter hypothesis. Moreover, the three effects of lump-sum income at the aggregate level are sufficient to explain the flypaper effect. Since they may have counterbalancing signs, the net effect of lump-sum income on public expenditures will normally be smaller than the positive effect of intergovernmental transfers. Greater lump-sum income may lead to a positive private income affect, but it may also lead to a negative substitution effect if it reduces marginal tax collections and the marginal tax base, and to a negative public income effect if reduces the amount of revenues.

4. Conclusions

This paper shows, formally and diagrammatically, that a change in lump-sum (private) income has three distinguishable effects on the tax and expenditures decisions of the government. This is because a change of lump-sum income leads to behavioral responses that alter the tax base and thus also the marginal cost of tax revenues, variable that becomes endogenous to the government maximization problem. This is the key aspect of the model developed in this paper and the main difference with respect to current approaches to the problem of determining optimal tax and expenditure policies.

The three effects of lump-sum income on government expenditures are: 1) the net substitution effect, which represents a change in the demand for public goods and services due to the change in the MCF; 2) the private income affect, a demand change due to greater consumption of private goods; and 3) the public income effect, which adjusts both the demand for public goods and services and the MCF function. As long as intergovernmental transfers do not directly alter taxpayers' behavior, they are shown to embody a pure public income effect on demand, and a subsidy of supply, but not a price effect as previously suggested in the literature.

It follows that alternative sources of private income lead to different marginal costs and income effects as they impact the budget constraint of the government differently. As a result, it is not correct to claim that all sources of income are equivalent, or that they can be summarized at the aggregate level by a unique income effect. When transfers are given to the government they may have no direct effects on tax collection costs, thus the recipient government might easily be able to reduce both the tax rate and the marginal costs of public expenditures. In contrast, when the same amount of transfers is given to the taxpayers the additional income is first available to finance private consumption, and in order to increase the amount of public goods and services the government will likely have to face higher marginal costs.

Like Dahlby (2011) we conclude that the flypaper effect can be explained as a consequence of the welfare maximizing behavior of a benevolent government. The proposed explanation is derived within the traditional framework of the social maximization problem, but it differs from other explanations currently available in the literature, including that of the fiscal illusion hypothesis, arguably the most accepted theory explaining the flypaper effect. Similar to the fiscal illusion hypothesis, we conclude that when lump-sum transfers are directly allocated to the sub-national government, economic agents inside the jurisdiction underestimate the true marginal costs of public expenditures. Indeed, the transfers received by the government were previously collected at a certain cost that is not considered locally. Different from the fiscal illusion hypothesis, however, this result is shown not to be the result of any form of taxpayers' "confusion." Instead, the proposed explanation suggests that lump-sum transfers to the local government do in fact help to reduce the marginal costs of public expenditures inside the jurisdiction. The problem, if any, is not the perceived cost of public expenditures, but instead that transfers embody a public income effect that may reduce the marginal cost of funds for any given level of public expenditures.

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Appendix I: Derivation of (4)

Applying the implicit function theorem to condition (2), the effect of z on the optimal tax rate t^* is equal to

$$N\frac{dt^*}{dz} = -\frac{1}{d^2\Omega/dt^2}\{X_{tz}^* u_x + R_{tz}^* N u_G + X_z^* X_t^* u_{xx} + R_z^* R_t^* N u_{GG}\}$$

$$= -\frac{1}{d^2\Omega/dt^2}\{[-B_z^* + (1-t)B_{tz}^*]u_x + (B_z^* + tB_{tz}^*)N u_G + X_z^* X_t^* u_{xx} + R_z^* R_t^* N u_{GG}\}$$

$$= -\frac{1}{d^2\Omega/dt^2}\{[B_{tz}^* - (B_z^* + tB_{tz}^*)]u_x + (B_z^* + tB_{tz}^*)N u_G + X_z^* X_t^* u_{xx} + R_z^* R_t^* N u_{GG}\}$$

$$= -\frac{1}{d^2\Omega/dt^2}\{(B_{tz}^* - R_{tz}^*)u_x + R_{tz}^* N u_G + X_z^* X_t^* u_{xx} + R_z^* R_t^* N u_{GG}\}$$

$$= -\frac{1}{d^2\Omega/dt^2}\{R_{tz}^*(N u_G - u_x) + B_{tz}^* u_x + X_z^* X_t^* u_{xx} + R_z^* R_t^* N u_{GG}\}$$

$$= -\frac{1}{d^2\Omega/dt^2}\left\{\left[R_{tz}^*\left(N\frac{u_G}{u_x} - 1\right) + B_{tz}^*\right]u_x + X_z^* X_t^* u_{xx} + R_z^* R_t^* N u_{GG}\right\} ;$$

finally, substituting (3) into this result we obtain (4).

Appendix II: Derivation of (5) and (6)

Given that the budget constraint of the government, $G = tB + S$, must be satisfied at the optimal solution choice of t^*, and that the latter implicitly optimizes the value of G, then the effect of S in G^* can be expressed as:

$$\frac{dG^*}{dS} = \frac{dt^*}{dS}B^* + tB_t^*\frac{dt^*}{dS} + 1 = \frac{dt^*}{dS}R_t^* + 1 , \qquad \text{(A.II.1)}$$

which is equal to (5). Moreover, using the same procedure the effect of Nz on G^* is equal to

$$N\frac{dG^*}{dz} = N\frac{dt^*}{dz}B^* + tB_t^* N\frac{dt^*}{dz} + tB_z^* = N\frac{dt^*}{dz}R_t^* + tB_z^* .$$

Using (4), this result can be written as

$$N\frac{dG^*}{dz} = \frac{\left[R_{tz}^*\left(N\frac{u_G}{u_x} - 1\right) + B_{tz}^*\right]u_x}{(-d^2\Omega/dt^2)}R_t^* + \frac{X_z^* X_t^* u_{xx}}{(-d^2\Omega/dt^2)}R_t^* + R_z^*\frac{dt^*}{dS}R_t^* + tB_z^* .$$

Finally, considering that $R_z^* = tB_z^*$ and using (A.II.1) we can easily obtain (6).

Appendix III: Derivation of (8) –concavity of the effective budget constraint

The slope of the effective budget constraint $m(t)$ as defined in (7) is:

$$m(t) = \frac{X_t^*}{R_t^*} = \frac{-B + (1-t)B_t^*}{B + tB_t^*} .$$

The derivative of $m(t)$ with respect to t is equal to:

$$\frac{dm(t)}{dt} = \frac{X^*_{tt}R^*_t - X^*_t R^*_{tt}}{{R^*_t}^2}$$

$$= \{[-2B^*_t + (1-t)B^*_{tt}]R^*_t - X^*_t(2B^*_t + tB^*_{tt})\}/{R^*_t}^2$$

$$= \{[-2B^*_t + (1-t)B^*_{tt}](B^* + tB^*_t) - [-B + (1-t)B^*_t](2B^*_t + tB^*_{tt})\}/{R^*_t}^2$$

$$= \{[\cancel{-2B^*_t} + (1-t)B^*_{tt}](B^* + tB^*_t) + (B + tB^*_t)[\cancel{2B^*_t} + tB^*_{tt}] - B^*_t(2B^*_t + tB^*_{tt})\}/{R^*_t}^2$$

$$= \{(1\cancel{-t})B^*_{tt}(B^* + tB^*_t)\cancel{+tB^*_{tt}(B + tB^*_t)} - B^*_t(2B^*_t + tB^*_{tt})\}/{R^*_t}^2$$

$$= \{B^*_{tt}(B^*\cancel{+tB^*_t}) - B^*_t(2B^*_t\cancel{+tB^*_{tt}})\}/{R^*_t}^2$$

$$= (B^*B^*_{tt} - 2B^{*2}_t)/{R^*_t}^2$$