Multidimensional poverty targeting

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Abstract

The importance of taking into account the multiple dimensions of wellbeing in the measurement of poverty has been recognized in the recent poverty measurement literature. The poverty alleviation literature has not, however, yet addressed the important issue of policy design for efficient multidimensional poverty reduction. To do this requires an integration of both 'normative' and 'positive' features. From a normative perspective, it is reasonable to argue that, in addition to being concerned with impacts on dimensional poverty, multidimensional poverty policy should also take into account impacts on the *joint* distribution of the various dimensions of poverty. From a positive perspective, the different dimensions of poverty often mutually reinforce each other, especially in cases of severe poverty over a relatively long period. The paper integrates these two perspectives into a consistent policy evaluation framework, considering in particular how poverty targeting may have a triple effect on multidimensional poverty: a direct effect on the targeted dimension, an indirect effect on joint deprivation, and a spill-over effect on the level of the other dimensions. Targeting dominance techniques are also proposed. The results are applied to data from Vietnam and South Africa and demonstrate the role of both normative and positive perspectives in setting efficient multidimensional poverty targeting policies.

Keywords: Targeting; Multidimensional poverty; Efficient policy; Vietnam; South Africa.

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1 Introduction

It is increasingly agreed that, to assess welfare in a multidimensional framework, it is important to take into account the interactions across the relevant welfare dimensions. This matters both for identifying the multidimensional poor (see for instance Alkire and Foster 2011) and for measuring the magnitude of their poverty (as in Bourguignon and Chakravarty 2003 for example). The interactions across the dimensions can occur in various manners, and can feed each other in the short and in the longer term. For instance, one person's individual attributes might be jointly determined with the attributes of his household or community (such as when a child's chances of survival depend on parental and community characteristics), external attributes that may also affect the individual's well-being. In many situations, the welfare attributes are positively correlated (*i.e.*, a deterioration in income worsens nutrition, and worse nutrition diminishes household productivity); however, negative correlations can also occur, as when an increase in child school attendance decreases household income, at least in the short term. The interactions between dimensions of well-being can be especially strong in cases of severe deprivation over a long period, as in the case of "continuing multi-dimensional poverty traps" (Thorbecke 2005). In the absence of appropriate policy, it may well be that the prevalence of multiple forms of deprivation and greater correlations across dimensions of well-being will increase over time, at the cost of greater multidimensional poverty.

The importance of taking into account multidimensional linkages in achieving poverty objectives has also been recognized. It is important to note, however, that means and objectives can easily be confused in a multidimensional poverty policy debate. For instance, although reducing uni-dimensional monetary poverty is often the salient policy objective, it is regularly achieved through multiple means and proxies. In many Latin American countries, for instance, conditional cash transfers (CCT) are allocated to low-income families using multidimensional proxies to estimate eligibility. Furthermore, the effects of those monetary transfers affect dimensions of well-being other than income or consumption, such as education and health. Multidimensional conditionality rules are also imposed to leverage as much as possible the cross-dimension effects of these cash transfers. The primary social objective of such rules and transfers, however, is typically not set explicitly in terms of a multidimensional poverty criterion.[1]

[1] One exception is the *Chile Solidario* program, which has the explicit objective of reducing multidimensional poverty (Fiszbein and Schady 2009).

To assert clearly whether the policy objective is in terms of multidimensional or uni-dimensional poverty reduction would seem important. As pointed out by Azevedo and Robles (2010), erroneously focussing policy on uni-dimensional poverty measurement runs the risk of leading to a sub-efficient fall in multidimensional poverty. (This is in fact one of the core messages of this paper.) Using multidimensional poverty indices to design policies to reduce uni-dimensional poverty can also be non-efficient. This is a point made by Ravallion (2011), who argues for instance that, to reduce income poverty, it is better to target the income poor, and that to reduce deprivation in access to public services, it is analogously better to target independently those that are deprived of such services. Using a multidimensional index of poverty (MIP) that mixes up the two dimensions would lead to a sub-efficient reduction of unidimensional income and public services poverty:

> "The total impact on (*multidimensional*) poverty would be lower if one based the allocation on the MIP [multidimensional index of poverty] rather than the separate poverty measures — one for incomes and one for access to services. It is not the aggregate index that we need for this purpose but its components." (Ravallion 2011, p. 240, our emphasis)

Unlike in Ravallion (2011), this paper assumes that the policy objective is to reduce multidimensional poverty and not to reduce poverty independently in each dimension (as is meant by the italicized term in the citation above). It has been well known for some time, however, that the efficient indicator to reduce a poverty index is not necessarily the poverty index itself — see for instance Kanbur (1987) and Besley and Kanbur (1988) for unidimensional poverty reduction.[2] We are not aware of previous work that derives efficient policy rules in order to reduce poverty in a formal multidimensional setting. The main goal of this paper is to do this, by setting the social objective function in terms of multidimensional poverty reduction, and by taking into account both the empirical, the normative and the spill-over effets of policy on the joint distribution of dimensions of well-being.

The paper thus considers the interdependencies of policy effects across multiple deprivations, as advocated in the 2009 Report of the Commission on the Measurement of

[2]Referring to their multidimensional poverty index (MIP), Alkire and Santos (2010) suggest that it "could be used to target the poorest, track the Millennium Development Goals, and design policies that directly address the interlocking deprivations poor people experience." (p. 1). Although the intention is clear (to reduce a MIP), it is unclear how the MIP itself can be of direct policy use. Rather, it would seem that explicit policy rules need to be derived in order to reduce efficiently the MIP. As will be seen later, these rules are not always straightforward transformations of a MIP.

Economic Performance and Social Progress (see Stiglitz, Sen, and Fitoussi 2009):

> "[T]he consequences for quality of life of having multiple disadvantages far exceed the sum of their individual effects. Developing measures of these cumulative effects requires information on the 'joint distribution' of the most salient features of quality of life across everyone in a country through dedicated surveys. (...) When designing policies in specific fields, impacts on indicators pertaining to different quality-of-life dimensions should be considered jointly, to address the interactions between dimensions and the needs of people who are disadvantaged in several domains." (pp. 15-16)

For expositional simplicity, the paper focuses on bi-dimensional poverty, although the insights and results can be extended to more than two dimensions. The paper then proceeds as follows. Section 2 presents the bi-dimensional poverty indices used in this study. It also explains how these bi-dimensional indices can be used to obtain a robust assessment of where poverty is greatest. This is done by building dominance surfaces based on intersection headcount indices, thus providing a justification for a focus in this paper on such intersection indices. The links between multidimensional intersection indices and poverty dominance are used later on to provide targeting policies that can be judged to be preferable over a wide set of procedures for measuring multidimensional poverty.

Section 2.3 discusses the (theoretical) impact on bi-dimensional poverty of targeting one dimension, for stylized additive and multiplicative transfers. Section 2.4 derives conditions for determining which population subgroup should be targeted first in order to obtain the largest population poverty reduction. Section 2.5 enriches these results by allowing for inter-dimensional spill-over effects.

Such theoretical results and their robustness are then tested in Section 3 with data from Vietnam (1992-1993 and 1997-1998) and South Africa (1993). Interesting insights emerge. It is found, for instance, that rules to decentralize geographical targeting funds may differ according to whether it is uni-dimensional or multidimensional poverty that national authorities intend to reduce. The nature of efficient allocation rules across socio-economic groups is also influenced by the class of poverty indices that are the objects of policy objectives as well as by the type of transfers that are envisaged. Section 4 concludes.

2 Framework

2.1 Measurement

It is one thing to concur that poverty is multidimensional; it is another to agree on a specific procedure to measure it. The literature has been building up a stock of various multidimensional indices over the recent years; see for instance Chakravarty, Mukherjee, and Ranade (1998), Tsui (2002), Bourguignon and Chakravarty (2003), and Alkire and Foster (2011). All such indices have the potential to order the extent of poverty differently across distributions. This also means that they may provide different policy guidelines, especially regarding the design of targeting schemes.

One way to circumvent this problem is to seek unanimity of policy guidance across classes of poverty measurement procedures. To do this, we follow the measurement framework of Duclos, Sahn, and Younger (2006) (DSY, for short), which we now briefly summarize. DSY starts by defining well-being (measured, for expositional simplicity, over two dimensions of well-being, x and y) as a function $\phi(x, y)$ that increases in both x and y. An unknown poverty frontier $\phi(x, y) = 0$ that separates the poor from the rich is then supposed to exist, a frontier over which individual well-being is equal to a "poverty level" of well-being, and below which individuals are in poverty. The set of the poor is then given by $\Lambda(\phi) = \{(x, y) \,|(\phi(x, y) \leq 0\}$. Multidimensional additive poverty indices can then be represented by

$$P(\phi) \; = \; \int \int_{\Lambda(\phi)} \pi(x, y; \phi) \, dF(x, y), \tag{1}$$

where $\pi(x, y; \phi)$ is the contribution to poverty of an individual with well-being indicators x and y and where $F(x, y)$ is the joint distribution of x and y.

DSY then defines a first-order class $\Pi^{1,1}(\phi^*)$ of bi-dimensional poverty indices. The indices that belong to that class must consider as potentially poor only those individuals that belong to the largest reasonable poverty set, defined by $\Lambda(\phi^*)$. The indices must also be continuous along the poverty frontier, be weakly decreasing in x and in y, and be such that the marginal poverty benefit of an increase in either x or y decreases with the value of the other variable. Atkinson and Bourguignon (1982) refer to this latter property as a property of non-decreasing poverty under a "correlation-increasing switch"; this implies that, *ceteris paribus*, the greater the incidence of multiple deprivation, the higher the level of multidimensional poverty.

Higher-order classes of poverty indices are obtained by imposing further assumptions on the derivatives of $\pi(x, y; \phi)$. For instance, the class $\Pi^{2,2}(\phi^*)$ of second-order indices are convex in x and in y; furthermore, that degree of complexity decreases with the level of the other indicators and at a decreasing rate. Further details can be found in DSY.

To test for whether the poverty ranking of two distributions is robust across all members of one such class of poverty indices, DSY introduces the following bi-dimensional poverty indices:

$$P(\alpha_x, \alpha_y) = \int_0^{z_x} \int_0^{z_y} \left(\frac{z_x - x}{z_x} \right)^{\alpha_x} \left(\frac{z_y - y}{z_y} \right)^{\alpha_y} dF(x, y) \tag{2}$$

where $\alpha_x \geq 0$, $\alpha_y \geq 0$, z_x and z_y are poverty lines in dimensions x and y respectively, and where $\left(\frac{z_x - x}{z_x}\right)$ and $\left(\frac{z_y - y}{z_y}\right)$ are called normalized "poverty gaps" in the poverty literature, respectively, in x and in y. Tracing (2) over areas of values of z_x and z_y draws a "dominance surface".

DSY then shows that if $P_A(\alpha_x, \alpha_y)$ for some distribution A is greater than $P_B(\alpha_x, \alpha_y)$ for some distribution B over all choices of (z_x, z_y) within $\Lambda(\phi^*)$, then poverty will be unambiguously higher in A than in B for all of the poverty indices that are members of the class $\Pi^{\alpha_x+1,\alpha_y+1}(\phi^*)$ of multidimensional poverty indices of order $(\alpha_x + 1, \alpha_y + 1)$ and for all poverty frontiers that lie within $\Lambda(\phi) \subset \Lambda(\phi^*)$. Note that these classes of indices include intersection, union, and intermediate poverty indices, as long as these fit within $\Lambda(\phi^*)$, although the index in (2) is an intersection index. The converse is also true: only if $P_A(\alpha_x, \alpha_y)$ is larger than $P_B(\alpha_x, \alpha_y)$ over all values of (z_x, z_y) within $\Lambda(\phi^*)$ can we be certain that poverty is unambiguously larger in A over all members of the class $\Pi^{\alpha_x+1,\alpha_y+1}(\phi^*)$ of multidimensional poverty indices of order $(\alpha_x + 1, \alpha_y + 1)$.

It cannot be argued convincingly that the intersection index in (2) is necessarily better than all other possible multidimensional poverty indices. The superiority of one index over another is generally a matter of value judgment. There are, however, important advantages in focusing on (2). (2) it is a natural generalization of the popular uni-dimensional FGT indices — see Foster, Greer, and Thorbecke (1984) — defined as

$$P(\alpha_x) = \int_0^{z_x} \left(\frac{z_x - x}{z_x} \right)^{\alpha_x} dF(x) \tag{3}$$

for poverty in x. Through its intersection nature, (2) also focuses on the poorest of the poor, that is, on those that suffer from multiple deprivation. Perhaps most importantly, if some policy consistently lowers (2) for a wide range of intersection poverty lines, then, by the result above, that policy will also reduce poverty for a large class of other poverty indices, possibly with different poverty frontiers. Such a result is unfortunately not available when using other types of multidimensional poverty indices.

2.2 Changes in poverty

Much of the paper rests on how (2) changes when dimensional indicators vary through policies and shocks. To do this, it is useful to extend (2) to cases in which α_x or α_y may be equal to minus one. Let then

$$P(\alpha_x = -1, \alpha_y) = f(z_x) \int_0^{z_y} \left(\frac{z_y - y}{z_y} \right)^{\alpha_y} f(y|\, x = z_x)\, dy, \tag{4}$$

where $f(z_x)$ is the density of x and $f(y|\, x)$ is the density of y conditional on x. $P(\alpha_x = -1, \alpha_y)$ is thus y-dimension FGT poverty of those individuals whose x value borders the x-dimension poverty line, times the density of those individuals in the population. Similarly,

$$P(\alpha_x, \alpha_y = -1) = f(z_y) \int_0^{z_x} \left(\frac{z_x - x}{z_x} \right)^{\alpha_x} f(x|\, y = z_y)\, dx. \tag{5}$$

It is also useful to rewrite $P(\alpha_x, \alpha_y)$ in a way that shows explicitly the role of attribute correlation in the valuation of multidimensional poverty. Knowing that

$$\begin{aligned} &\operatorname{cov}\left[\left(\tfrac{z_x - x}{z_x}\right)_+^{\alpha_x}, \left(\tfrac{z_y - y}{z_y}\right)_+^{\alpha_y} \right] \\ &= E\left[\left(\tfrac{z_x - x}{z_x}\right)_+^{\alpha_x} \left(\tfrac{z_y - y}{z_y}\right)_+^{\alpha_y} \right] - E\left[\left(\tfrac{z_x - x}{z_x}\right)_+^{\alpha_x} \right] E\left[\left(\tfrac{z_y - y}{z_y}\right)_+^{\alpha_y} \right], \end{aligned} \tag{6}$$

where $f_+ = \max(f, 0)$, we can rewrite (2) as:

$$\begin{aligned} &P(\alpha_x, \alpha_y) \\ &= P(\alpha_x)P(\alpha_y) + \operatorname{cov}\left[\left(\tfrac{z_x - x}{z_x}\right)_+^{\alpha_x}, \left(\tfrac{z_y - y}{z_y}\right)_+^{\alpha_y} \right]. \end{aligned} \tag{7}$$

Thus, bi-dimensional poverty $P(\alpha_x, \alpha_y)$ equals the product of the two unidimensional poverty indices plus the covariance between the poverty gaps in the two attributes. This latter term captures the importance of the "correlation" between the two dimensions.

Duclos, Sahn, and Younger (2006) illustrates how this covariance term can play a crucial role in multidimensional poverty dominance. It can happen for instance that urban areas unidimensionally dominate rural areas both in income and in nutrition, but not bi-dimensionally, because urban areas display greater levels of multiple deprivation. It can also happen that, although unidimensional comparisons may be ambiguous, multidimensional comparisons are not, the ambiguity being resolved by the joint distribution information.

More generally, inspection of (7) shows why a policy focus on unidimensional poverty ($P(\alpha_x)$, say) may lead to different guidance from a focus on multidimensional poverty. Not only does $P(\alpha_y)$ multiply $P(\alpha_x)$, but the covariance of multiple deprivation also distinguishes $P(\alpha_x)$ from $P(\alpha_x, \alpha_y)$. The policy effects of this difference are now considered. In section 2.5, an additional distinction is introduced by considering cases in which transfers in the x dimension have "spill-over" effects on the y dimension.

2.3 The effect of one-dimension targeting

We now consider how changes in either dimension can affect multidimensional poverty. These changes can come from different sources, such as growth and macroeconomic shocks. We focus on the impact of targeting policies, although the results are extendable to other sources of distributional changes.

2.3.1 Additive transfers

Assume that an additive transfer γ is granted to everyone in a population. This is a simplifying framework; it will be enriched later on in the paper. We can then re-write (2) as

$$P(\alpha_x, \alpha_y, \gamma) = \int \int \left(\frac{z_x - x - \gamma}{z_x} \right)_+^{\alpha_x} \left(\frac{z_y - y}{z_y} \right)_+^{\alpha_y} dF(x, y) \tag{8}$$

and also express $P(-1, \alpha_y, \gamma)$ and $P(\alpha_x, -1, \gamma)$ in (4) and (5) analogously. For $\alpha_x > 0$, a marginal change in γ will change the bi-dimensional poverty index (2) by

$$\left.\frac{\partial P(\alpha_x, \alpha_y, \gamma)}{\partial \gamma}\right|_{\gamma=0} = \frac{\alpha_x}{z_x} P(\alpha_x - 1, \alpha_y)$$

$$= -\frac{\alpha_x}{z_x} P(\alpha_x - 1) P(\alpha_y) - \frac{\alpha_x}{z_x} \operatorname{cov}\left[\left(\frac{z_x - x}{z_x}\right)_+^{\alpha_x - 1}, \left(\frac{z_y - y}{z_y}\right)_+^{\alpha_y}\right]. \tag{9}$$

$P(\alpha_x - 1)$ in (9) is the impact of targeting on unidimensional poverty identified in Kanbur (1985). It also corresponds to the well-known result that the sensitivity of unidimensional FGT poverty to changes in welfare is related to the same FGT index, but with parameter set to $\alpha - 1$. For multidimensional poverty, this effect must be multiplied by the level of unidimensional poverty in the other dimension — the term $P(\alpha_y)$ in (9) — although this other dimension is not targeted by the transfer. The multidimensional poverty impact must also incorporate the covariance between the poverty gaps in the dimensions x and y, to the powers $\alpha_x - 1$ and α_y. As we will see later in the illustration, these additional effects can lead to different unidimensional and multidimensional policy prescriptions.

If $\alpha_x = 0$, the change in multidimensional poverty is given by:

$$\left.\frac{\partial P(\alpha_x, \alpha_y, \gamma)}{\partial \gamma}\right|_{\gamma=0} = -P(\alpha_x = -1, \alpha_y). \tag{10}$$

The headcount impact of targeting is thus proportional to the density of individuals around z_x times the unidimensional FGT index in dimension y, for those at $x = z_x$. The impact of targeting on the intersection bi-dimensional headcount is therefore quite different from the value of the intersection headcount itself. It can also differ significantly from the headcount index in the x dimension.

The *per capita* cost of a universal additive transfer is $R(\gamma) = \gamma$, with $\partial R(\gamma)/\partial \gamma = 1$. The change in aggregate poverty per additional dollar spent *per capita* is thus also given by (9) and (10).

2.3.2 Multiplicative transfers

An alternative and commonly-modeled form of targeting increases a pre-transfer indicator x by some proportion λ. (The poverty impact of growth in x can be similarly modeled.) Algebraically, post-transfer poverty can be written as

$$P(\alpha_x, \alpha_y, \lambda) = \int\int \left(\frac{z_x - x(1+\lambda)}{z_x}\right)_+^{\alpha_x} \left(\frac{z_y - y}{z_y}\right)_+^{\alpha_y} dF(x, y). \tag{11}$$

When $\alpha_x > 0$, the derivative of (11) with respect to λ can be shown to be given by

$$\frac{\partial P(\alpha_x, \alpha_y, \lambda)}{\partial \lambda} = -\frac{\alpha_x}{(1+\lambda)}[P(\alpha_x - 1, \alpha_y) - P(\alpha_x, \alpha_y)]. \tag{12}$$

The *per capita* cost of such a multiplicative transfer is

$$R(\lambda) = \lambda \overline{x}, \tag{13}$$

where $\overline{x}$ is the average of x. The change in aggregate poverty per dollar spent *per capita* is then:

$$\left.\frac{\partial P(\alpha_x, \alpha_y, \lambda)}{\partial \lambda} \middle/ \frac{\partial R(\lambda)}{\partial \lambda}\right|_{\lambda=0} = -\frac{\alpha_x}{\overline{x}}[P(\alpha_x - 1, \alpha_y) - P(\alpha_x, \alpha_y)]. \tag{14}$$

The expression above is always negative since $P(\alpha_x - 1, \alpha_y) > P(\alpha_x, \alpha_y)$ for $\alpha_x > 0$. (14) is complex since it involves comparing the value of two bi-dimensional indices. Poverty reduction following a multiplicative transfer is faster the greater the difference between $P(\alpha_x - 1, \alpha_y)$ and $P(\alpha_x, \alpha_y)$. Intuitively, this occurs when multiplicative transfers decrease the poverty gaps of the poor fast. This requires the x values of the poor to be not too close to 0.

If $\alpha_x = 0$, the change in the bi-dimensional headcount per dollar spent is

$$\left.\frac{\partial P(\alpha_x, \alpha_y, \lambda)}{\partial \lambda} \middle/ \frac{\partial R(\lambda)}{\partial \lambda}\right|_{\lambda=0} = -\frac{z_x}{\overline{x}} P(\alpha_x = -1, \alpha_y). \tag{15}$$

Comparing (9) to (14), and (10) to (15), it is not possible to say *a priori* whether for every *per capita* dollar spent, an additive transfer contributes more than a multiplicative transfer to multidimensional poverty reduction. When $\alpha > 0$, the answer depends on the values of $P(\alpha_x - 1, \alpha_y) - P(\alpha_x, \alpha_y)$ (the term that appears in the multiplicative case) and $P(\alpha_x - 1, \alpha_y)$ (which appears in the additive case). If the *per capita* multiplicative targeting cost were equal to z_x (with $z_x = \overline{x}$), then additive targeting would be more poverty effective: it would reduce poverty per dollar spent faster because it would give the poorest among the poor a greater increase in x. For relatively poor societies, where $\overline{x}$ is below and far from the poverty line z_x, a multiplicative transfer might yet be better if it pushes up fast the x values of the better off among the x poor.

As for bi-dimensional poverty with $\alpha_x = 0$, we need to compare z_x with $\overline{x}$. If z_x is greater than $\overline{x}$, then a multiplicative transfer reduces poverty faster. This will occur for very poor societies. Otherwise, additive targeting is preferred, regardless of the value of the multidimensional and unidimensional indices.

2.4 Socio-economic targeting

In addition to taking various forms (such as additive and multiplicative ones), targeting is rarely uniform across the population. Sociodemographic characteristics are in particular often used to target transfers, leading to "socio-economic targeting". We now turn to how we may rank the poverty alleviation efficiency of such socio-economic targeting schemes.

2.4.1 Additive transfers

Developing the framework above, we can provide insights as to which population subgroup should be targeted in order to reduce population poverty the most per dollar spent. For simplicity, assume that the total population is divided into two exclusive groups, A and B (such as urban and rural areas, or regions/provinces in the empirical illustrations below). Bi-dimensional poverty is then given by

$$P(\alpha_x, \alpha_y, \gamma^A, \gamma^B) = \omega^A P^A(\alpha_x, \alpha_y, \gamma^A) + \omega^B P^B(\alpha_x, \alpha_y, \gamma^B), \tag{16}$$

where ω^A and ω^B are the population shares of groups A and B, γ^A and γ^B are transfers targeted specifically to members of groups A and B, and P^A and P^B are poverty levels for groups A and B, respectively.

To assess whether, for efficient population-level poverty reduction, an additive transfer is better targeted towards group A or towards group B, we need to check whether

$$\frac{\partial P(\alpha_x, \alpha_y, \gamma^A)}{\partial \gamma^A} \bigg/ \frac{\partial R(\gamma^A, \gamma^B)}{\partial \gamma^A} \lesseqgtr \frac{\partial P(\alpha_x, \alpha_y, \gamma^B)}{\partial \gamma^B} \bigg/ \frac{\partial R(\gamma^A, \gamma^B)}{\partial \gamma^B}, \tag{17}$$

where the *per capita* cost of an additive transfer is given by

$$R = \omega^A \gamma^A + \omega^B \gamma^B. \tag{18}$$

We start with the case of $\alpha_x > 0$. We then have:

$$\frac{\partial P(\alpha_x, \alpha_y, \gamma^A)}{\partial \gamma^A} \Big/ \frac{\partial R}{\partial \gamma^A}\bigg|_{\gamma^A=\gamma^B=0} = -\frac{\alpha_x}{z_x} P^A(\alpha_x - 1, \alpha_y) \tag{19}$$

and, similarly,

$$\frac{\partial P(\alpha_x, \alpha_y, \gamma^B)}{\partial \gamma^B} \Big/ \frac{\partial R}{\partial \gamma^B}\bigg|_{\gamma^A=\gamma^B=0} = -\frac{\alpha_x}{z_x} P^B(\alpha_x - 1, \alpha_y). \tag{20}$$

The largest aggregate poverty reduction per dollar spent *per capita* is then obtained by targeting that group that has the highest $P(\alpha_x - 1, \alpha_y)$ index. Looking back to (9), note that this will be the case for the group that displays the highest $P(\alpha_x - 1)$ index, the largest $P(\alpha_y)$ index, and/or the highest covariance between $\alpha_x - 1$ and α_y uni-dimensional gaps. It is clear that simply choosing the group to target on the basis of the $P(\alpha_x)$ indices will generally not lead to efficient multidimensional poverty reduction strategies.

For $\alpha_x = 0$, $\frac{\alpha_x}{z_x} P^A(\alpha_x - 1, \alpha_y)$ and $\frac{\alpha_x}{z_x} P^B(\alpha_x - 1, \alpha_y)$ in (19) and (20) above are replaced respectively by $P^A(\alpha_x = -1, \alpha_y, \gamma^A)$ and $P^B(\alpha_x = -1, \alpha_y, \gamma^B)$. Again, the multidimensional poverty index itself is not the right guide to selecting the better group to target. Instead, the efficient targeting rule uses the y-dimension FGT index of those that are around the x poverty line, multiplied by the density of the group's individuals at the x-dimension poverty line.

2.4.2 Multiplicative transfers

Let us now assess the efficient group selection rule under a multiplicative targeting scheme. The *per capita* cost of such a scheme is given by

$$R = \omega^A \overline{x}^A + \omega^B \overline{x}^B \tag{21}$$

and, when $\alpha_x > 0$, changes in poverty due to a multiplicative transfer λ in groups A and B respectively are given by

$$\left.\frac{\partial P(\alpha_x, \alpha_y, \lambda^A)}{\partial \lambda^A} \Big/ \frac{\partial R}{\partial \lambda^A}\right|_{\lambda^A=\lambda^B=0} = -\frac{\alpha_x}{\overline{x}^A}[P^A(\alpha_x - 1, \alpha_y) - P^A(\alpha_x, \alpha_y)] \tag{22}$$

and

$$\left.\frac{\partial P(\alpha_x, \alpha_y, \lambda^B)}{\partial \lambda^B} \Big/ \frac{\partial R}{\partial \lambda^B}\right|_{\lambda^A=\lambda^B=0} = -\frac{\alpha_x}{\overline{x}^B}[P^B(\alpha_x - 1, \alpha_y) - P^B(\alpha_x, \alpha_y)]. \tag{23}$$

For $\alpha_x = 0$, these expressions become

$$\left.\frac{\partial P(\alpha_x, \alpha_y, \lambda^A)}{\partial \lambda^A} \Big/ \frac{\partial R}{\partial \lambda^A}\right|_{\lambda^A=\lambda^B=0} = -\frac{z_x}{\overline{x}^A}P^A(\alpha_x = -1, \alpha_y) \tag{24}$$

and

$$\left.\frac{\partial P(\alpha_x, \alpha_y, \lambda^B)}{\partial \lambda^B} \Big/ \frac{\partial R}{\partial \lambda^B}\right|_{\lambda^A=\lambda^B=0} = -\frac{z_x}{\overline{x}^B}P^B(\alpha_x = -1, \alpha_y). \tag{25}$$

Again, the case in which the transfer is a proportion of dimension x is less straightforward to interpret than the case of an additive transfer. Looking back to (22) and (23), the reduction in multidimensional poverty per dollar spent is the largest for those groups with the lowest average income and the greatest distance between $P(\alpha_x - 1, \alpha_y)$ and $P(\alpha_x, \alpha_y)$. Those groups living in more deprived conditions in dimension x will have a lower $\overline{x}$ but will not necessarily show a tendency for poverty gaps to decrease faster with an increase in α, although the difference in poverty gaps of order $\alpha - 1$ and α is likely to be larger for such groups. In addition, those groups are also likely to show higher poverty in other dimensions, but not necessarily so; the assessment must also take into account the correlation across dimensions (recall (7)).

For $\alpha_x = 0$, multidimensional population poverty falls fastest per dollar spent when targeting favors those groups whose $P(-1, \alpha_y)$ is largest and/or whose average income is lowest, the explicit trade-off being shown in (24). A large $P(-1, \alpha_y)$ value is observed when the density around the x poverty line is large, and/or when those around that poverty line have a large y poverty gap of order α_y.

2.5 Targeting with dimensional spill-overs

Now suppose that dimension y is also indirectly affected by transfers γ made to dimension x. We suppose that this indirect — or spill-over — effect on y is captured by a function $\sigma(y,\gamma)$, which is equal to y in the absence of spill-over effects and with $\sigma(y,0)=y$. We may thus re-write (8) as

$$P(\alpha_x,\alpha_y,\gamma)=\int\int\left(\frac{z_x-x-\gamma}{z_x}\right)_+^{\alpha_x}\left(\frac{z_y-\sigma(y,\gamma)}{z_y}\right)_+^{\alpha_y}dF(x,y). \tag{26}$$

For expositional purposes, let us think of x and y as income and nutrition (or health), respectively, two dimensions in which welfare analysts are often jointly interested. (26) shows that a policy that targets income explicitly (for instance, through a cash transfer) affects multidimensional poverty directly through its impact on the poverty gap in dimension x, through its multiplying effect on the gap in the other dimension y, and through its spill-over effect on that other dimension, captured in (26) by $\sigma(y,\gamma)$.

For $\alpha_y>0$, the marginal *spill-over effect* on bi-dimensional poverty of a change in γ is then given by

$$\begin{aligned}\left.\frac{\partial P(\alpha_x,\alpha_y,\gamma)}{\partial\gamma}\right|_{\text{spillover effect},\gamma=0} &= -\frac{\alpha_y}{z_y}P(\alpha_x)\int_0^{z_y}\left.\frac{\partial\sigma(y,\gamma)}{\partial\gamma}\right|_{\gamma=0}\left(\frac{z_y-y}{z_y}\right)^{\alpha_y-1}dF(y) \\ &+\frac{\alpha_y}{z_y}\operatorname{cov}\left[\left(\frac{z_x-x}{z_x}\right)_+^{\alpha_x},\left.\frac{\partial\sigma(y,\gamma)}{\partial\gamma}\right|_{\gamma=0}\left(\frac{z_y-y}{z_y}\right)_+^{\alpha_y-1}\right],\end{aligned} \tag{27}$$

and, for $\alpha_y=0$, by

$$\begin{aligned}&\left.\frac{\partial P(\alpha_x,\alpha_y,\gamma)}{\partial\gamma}\right|_{\text{spillover effect},\gamma=0}= \\ &-\left.\frac{\partial\sigma(y,\gamma)}{\partial\gamma}\right|_{y=z_y,\gamma=0}f(\sigma(y,\gamma)=z_y)\int_0^{z_x}\left(\frac{z_x-x}{z_x}\right)^{\alpha_x}dF\left(x|\,y=z_y\right).\end{aligned} \tag{28}$$

This spill-over effect adds to the other effects described above, either through the impact of an additive or of a multiplicative transfer on dimension x. For instance, the net multidimensional poverty effect of an additive transfer to dimension x would be the sum of (9) (or (10) for $\alpha_x=0$) and either (27) or (28). For a multiplicative transfer, expression (9) is replaced by (12), and analogously for $\alpha_x=0$.

The formulation of $\sigma(y,\gamma)$ is sufficiently general to allow for several types of spill-over effects on the second dimension. Special cases include additive spillover effects, when $\sigma(y,\gamma) = y + \gamma$, or multiplicative ones, when $\sigma(y,\gamma) = (1+\gamma)y$. In all cases, the spill-over effect is given by the mean of the product of the y poverty gaps to the power $\alpha - 1$ and the marginal change in $\sigma(y,\gamma)$, weighted by the x poverty gaps to the power α_x.

Whether this indirect effect favors targeting the more severely poor depends on whether the severely poor's welfare indicator y is more sensitive to γ. That may or may not be the case. It also depends on whether the more severely poor in the x dimension are also poor in the y dimension, which again may or may not be the case.

These spillover effects can then be normalized by the *per capita* cost of targeting dimension x. This is done in the same way as in Section 2.4. Doing so makes it possible to assess which population subgroup should be targeted first in order to reduce multidimensional poverty as quickly as possible, subject to resource constraints. If a *per capita* targeting cost can also be assessed for each of the two dimensions, x and y, then such a normalization further allows establishing which *dimension* (in addition to which *group*) should preferably be targeted by public expenditures.

2.6 Targeting dominance

As in Section 2.1 for comparing poverty across two distributions, we might also want to ensure that our targeting conclusions and policy recommendations are robust to the choice of multidimensional poverty indices and to the choice of multidimensional poverty frontiers. As in Section 2.1, we can do this for classes of indices denoted by $\Pi^{\alpha_x+1,\alpha_y+1}(\phi^*)$. To test for whether a targeting preference for a group is robust to the choice of a multidimensional poverty index within one such class of poverty indices, we can use "targeting dominance surfaces". These surfaces are given by expressions such as (10), (14), (19), (22) and (27) over areas of intersection poverty frontiers (z_x, z_y). For instance, to rank robustly the impact of additive and proportional transfer policies over the class $\Pi^{1,1}(\phi^*)$ of multidimensional poverty indices, the targeting dominance surfaces given by (10) and (15) are compared over an area of intersection poverty frontiers (z_x, z_y) lying within $\Lambda(\phi^*)$. Formally, assuming no spillover effect:

Proposition 1

For all $P(\phi) \in \Pi^{1,\alpha_y+1}(\phi^*)$ *and for* $\gamma = \lambda\overline{x}$*,* $P(\phi,\gamma) \leq P(\phi,\lambda)$ *for marginal* γ *and* λ *if and only if*

$$-P(\alpha_x = -1, \alpha_y) \leq -\frac{z_x}{\overline{x}} P(\alpha_x = -1, \alpha_y)\, \forall (z_x, z_y) \in \Lambda(\phi^*). \tag{29}$$

This says that additive targeting will decrease poverty faster, *per capita* dollar spent, then multiplicative targeting for all indices of poverty in $\Pi^{1,\alpha_y+1}(\phi^*)$ if and only if expression (10) is always found to be lower than (15) regardless of the choice of intersection poverty frontiers, as long as these frontiers lie within the maximum domain of poverty frontiers within which a multidimensional poverty assessment can reasonably be made. As above, the dominance tests compare additive and multiplicative impacts on multidimensional *intersection* indices, although robustness is obtained over indices that include intersection, union, and intermediate poverty indices.

Extensions of Proposition 1 can be made straightforwardly by allowing for spillover effects, by considering higher classes of indices in dimension x ($\alpha_x + 1 > 1$), or by assessing whether robust socio-economic targeting conclusions can be obtained over classes of indices.

3 Illustrations

3.1 Multiple deprivation and dimensional spill-over effects

As discussed above, the correlation — and more generally, the joint distribution — of dimensions is important both for measurement and for policy purposes. From an empirical perspective, much of this correlation usually reflects a "natural" distribution of dimensions. An example is the relation between maternal nutrition and child weight at birth. Another is the correlation between child nutrition and schooling performance (and adult labor outcomes): child malnutrition (especially if experienced during the first two years of life) is usually associated with lower school and labor performance (see, for example, Glewwe and King 2001, Heckman 2008 and Alderman, Hoddinott, and Kinsey 2006 for discussion and evidence). Important direct and indirect costs (including opportunity costs) can also hinder the school attendance of the monetarily poor children, leading to class repetition and late or no enrollment.

Some of that joint distribution between indicators of well-being can be driven (at least partly) by policy. Policy can influence the multidimensional distribution of such indicators in a number of different ways. The subsidized or free provision of services such as education, health and housing may be one way to alleviate poverty in each of its multiple dimensions. Public investments in perinatal care (for instance, through pre-natal health visits and nutritional programs for pregnant women) can improve the health status of newborn children and their later life prospects. In the absence of affordable and good-quality public health services, those that are monetarily poor are also more likely to experience bad health conditions. Policy can thus serve to reduce the correlation across dimensional deprivation statuses and thus reduce the prevalence of multiple deprivations. This is true in the short term, although the effects may also be reinforced over time through the existence of multidimensional poverty traps.

Policies are indeed often designed in a way that attempts to address the multidimensionality of poverty. The popularized conditional cash transfer (CCT) programs intend for instance to break down the multidimensional (and multi-generational) poverty traps both by alleviating monetary poverty and by increasing human capital levels (health and education). A key mechanism that is employed is the multidimensional conditionality of the transfers.

The cross-dimension effects of transfer conditionality have been most extensively demonstrated in the context of Latin American countries. For example, Fiszbein and Schady (2009) show plenty of cross-country evidence of CCT's positive impacts on various health indicators and access to health services, school enrolment and attendance, and — most prominently because of the nature of the programs — on income poverty.

For instance, the effect on nutritional poverty of a cash transfer conditioned on family investments in child nutrition is likely to be higher than one without conditionality; the short-term effect on monetary poverty may, however, be reduced by conditionality, if, for instance, some of the transfers cannot then be used for purely income production purposes. Hence, conditionality may not be efficient for monetary poverty reduction, but may be best for reducing multidimensional poverty.

The correlation across attributes of well-being and the ability of policy to modify it also depend on the quality of markets. When markets do not exist or are highly imperfect, social programs may be less effective at producing positive spill-over effects on dimensions other than the targeted one. For example, in remote areas where appropriate schooling infrastructure is missing or is of poor quality, social cash transfers for children may have meagre effects on school outcomes (see for instance Kakwani, Soares, and Son 2006 and Cockburn, Fofana, and Tiberti 2010).

This suggests again the usefulness of a consistent multidimensional framework for assessing the impact of policy. It is not possible, of course, to take into account all of the possible effects of policy on the multiple dimensions of poverty. It is nevertheless feasible and, we believe, useful to apply the analytical framework developed above to illustrate how these effects can feed into policy design and policy evaluation. We do this in two different steps. We first assess the poverty impact and the efficiency of simple targeting rules established on the basis of socioeconomic characteristics, following the strong targeting tradition of the unidimensional poverty literature. We then enrich those simple rules by assessing more realistically the impact of policies, policies that can have spill-over effects beyond the dimensions that are targeted.

3.2 Data and estimation procedures

We apply the analytical approach presented above to three separate datasets from Vietnam and South Africa. These are the Vietnam Living Standard Survey (VLSS) 1992-1993, the VLSS 1997-1998 and the South Africa Integrated Household Survey (SAIHS) 1993. These three data sets include information on household consumption and anthropometric measures, which is a major reason for why we are using them. This information enables the construction of *per capita* household consumption (deflated by appropriate spatial and temporal price deflators) and height-for-age z scores (*HAZ*), standardized by the growth standards found in WHO (2006). These indicators of monetary welfare and of health are used for income poverty and health poverty respectively. The analysis focuses on children under five years old. It is supposed that policy can target *per capita* expenditure (dimension x in the above analytical framework), but that the outcome of that policy depends on its impact on the joint distribution of expenditure and *HAZ* (dimension y).

The spill-over effect on health of targeting expenditures is obtained by estimating the following simple regression model:

$$y_i = \alpha + \beta_x x_i + \sum_k \beta_k z_{k,i} + \epsilon_i, \tag{30}$$

where y_i is the z-score variable for child i, x_i is *per capita* consumption, β_x is the coefficient associated to *per capita* consumption, z_k is determinant k, β_k are their associated coefficients, and ϵ_i is the error term. The econometric model retained to estimate the spill-over coefficient is that proposed by Wagstaff, van Doorslaer, and Watanabe (2003), which uses OLS estimation with community-level fixed effects at the level of the child's commune. Note that the model is intended to provide a simple, reduced-form, representation of potentially complex mechanisms linking consumption to children's health. These mechanisms will generally depend on household composition and intra-household allocation rules, rules that are not observable to the analyst. An example is the distribution of cash transfers for the benefit of children. These can be directly distributed to adults, with a potentially diluted effect on the targeted children. The transfers can alternatively take the form of nutritional transfers, which could in principle be potentially better targeted to children; with these transfers, there exist, however, substitution strategies that parents can use in order to substitute away from children some of the additional resources intended for them.

Table 1 shows descriptive statistics on the *HAZ* as well as the explanatory variables appearing in the *HAZ* regressions. The estimated coefficients of the *HAZ* regressions are shown in Table 2. Most of the coefficients take the expected sign in all three surveys. *Per capita* real expenditures are positively associated with child health; child health is negatively (and convexly) linked to child age; in South Africa, being male is associated with worse nutrition, while having access to improved sanitation facilities improves nutrition statistically only in Vietnam 1992-93. Somewhat surprisingly, the estimated parameters on access to safe water sources and maternal schooling are not statistically significant.

The spill-over parameters that are produced by the estimates of Table 2 are 0.0171 percent for VLSS 1992-1993, 0.0097 percent for VLSS 1997-1998 and 0.1766 percent for SAIHS 1993. These parameters are obtained as ratios between $ln(pc\_consumption)$'s coefficients in Table 2 and the exponential mean of $ln(pc\_consumption)$. They are then calculated as $0.2470/\exp(7.2705)$ for VLSS 1992-1993, $0.1885/\exp(7.5709)$ for VLSS 1997-1998 and $0.2842/\exp(5.0808)$ for SAIHS 1993. These spill-over effects are used below in valuing the impact of a variation in household *per capita* income onto *HAZ* values for children.

3.3 Results

We proceed by separating the population into separate sub population geographical groups — see their definition in Table 3. As suggested in WHO (2006), out-of-range values (<-5 and >3) for the z-scores are dropped. For ease of exposition, a value of 10 is added to the *HAZ* variable and to the poverty lines in the health dimension; such a transformation does not affect any of the substantive results since we are interested in absolute multidimensional poverty, not relative multidimensional poverty or inequality.

For benchmarking purposes, a reference annual monetary poverty line of 1790 thousands Dong (in 1998 prices) is used for the two Vietnamese surveys, while a monthly

Table 1: Means and standard deviations of variables included in the *HAZ* regressions

	VLSS92-93	**VLSS97-98**	**SAIHS93**
HAZ	-2.20	-1.71	-1.22
	(1.35)	(1.33)	(1.47)
ln(pc_consumption)	7.27	7.57	5.08
	(0.52)	(0.57)	(0.92)
age_months	32.02	33.43	31.32
	(17.43)	(17.78)	(16.71)
age_months2	1328.86	1433.78	1260.27
	(1118.27)	(1167.18)	(1063.74)
gender	0.50	0.51	0.50
	(0.50)	(0.50)	(0.50)
safe_water	0.79	0.73	0.83
	(0.41)	(0.45)	(0.37)
safe_sanitation	0.14	0.20	0.35
	(0.35)	(0.40)	(0.48)
schooling_mother	6.51	2.73	5.56
	(3.44)	(0.99)	(3.61)
# of observations	2754	2195	3858

Note: standard deviations are reported in parenthesis. Means and standard deviations are estimated on the sample of children 0-5 years old retained for the regression analysis

Source: authors' analysis based on VLSS 1992-1993, VLSS 1997-1998 and SAIHS 1993

monetary poverty line of 164 Rand is used for South Africa. These values correspond to around 385 and 75 international dollars (in 2005 prices) respectively. For health, a poverty threshold of -2 standard deviations is used for each of the three countries — this threshold is often used to identify moderate-to-severe stunting (following the transformation of the *HAZ* variable, the reference health poverty threshold is set to 8). These poverty lines or used for reference purposes. For dominance, ranges of poverty lines are needed. For practical purposes, ten different poverty lines (equal to or lower than the reference poverty lines) for each of the two dimensions are used, yielding 100 possible combinations.

We focus on bi-dimensional poverty with $\alpha_x = \alpha_y = 0$ and $\alpha_x = \alpha_y = 1$, normalized by the *per capita* cost of the policy. The geographical units are ordered according to the importance of the marginal poverty reduction following a marginal increase of a cash

Table 2: *HAZ* regressions' coefficients

explanatory variables	**VLSS92-93**	**VLSS97-98**	**SAIHS93**
ln(pc_consumption)	0.2470	0.1885	0.2842
	(3.61)	(2.24)	(6.47)
age_months	-0.0764	-0.0652	-0.0567
	(-12.55)	(-9.6)	(-9.8)
age_months2	0.0010	0.0007	0.0008
	(10.88)	(7.38)	(8.53)
gender	0.0262	-0.0368	-0.1232
	(0.54)	(-0.67)	(-2.71)
safe_water	0.0543	0.0945	-0.1752
	(0.5)	(1.00)	(-1.72)
safe_sanitation	0.2405	0.1007	0.1404
	(2.68)	(1.12)	(0.95)
schooling_mother	0.0167	0.0043	0.0135
	(1.61)	(0.11)	(1.74)
constant	-3.0117	-2.0918	-1.7532
	(-6.27)	(-3.42)	(-7.14)
Adj. R^2	0.1551	0.2013	0.1696
# of observations	2754	2195	3858

Note: *t*-Stats are reported in parenthesis. Explanatory variables are not necessarily comparable across surveys since their definition can differ
Source: authors' analysis based on VLSS 1992-1993, VLSS 1997-1998 and SAIHS 1993

transfer. An important lesson is that those rankings change significantly once we move away from uni-dimensional towards multidimensional poverty alleviation.

We start with Vietnam 1992, using $\alpha_x = \alpha_y = 0$ and for the reference poverty lines mentioned above. We also first consider additive transfers. The results are shown in the upper panel A of Table 4. The upper panel A of Table 4 is split into four different sets of columns. The first set of columns shows the unidimensional results, namely, those results based only on the monetary impact of the transfer. The second set of columns adds to this the impact on the second dimension. The third set of columns incorporates the impact of the monetary transfer on the covariance of deprivation. The last set of columns shows the total multidimensional poverty impact of the transfer, adding to the earlier effects the spillover effect on the non-targeted dimension.

The very first column of Table 4 shows the priority ranking that must be assigned to

Table 3: Numbering of the geographical groups, VLSS92-93, VLSS97-98 and SAIHS93

(a) *VLSS92-93*		area	
		urban	rural
region	RedRiverDelta	8	6
	Northeast	3	3
	Northwest	4	4
	NorthCentralCoast	7	7
	SouthCentralCoast	5	10
	CentralHighlands	1	1
	Southeast	5	2
	MekongRiverDelta	9	2

(b) *VLSS97-98*		area	
		urban	rural
region	RedRiverDelta	5	1
	Northeast	10	10
	Northwest	3	3
	NorthCentralCoast	10	10
	SouthCentralCoast	7	2
	CentralHighlands	9	9
	Southeast	4	6
	MekongRiverDelta	8	2

(c) *SAIHS93*		area		
		metro	urban	rural
province	Western Cape	1	1	1
	Northern Cape	2	2	
	Eastern Cape	3	4	5
	KwaZulu-Natal	6	7	8
	Free State	9	10	
	Mpumalanga	11	12	
	Limpopo	13	14	
	North West	15	16	
	Gauteng	17	17	18

Note: The geographical groups appearing in the tables were obtained as a combination of regions/provinces and areas. *E.g.*, group "1" in *VLSS92-93* corresponds to the combination of Central Highlands region together with urban and rural areas.

the groups shown in the other columns. Focusing first on unidimensional poverty, a significantly larger reduction in the poverty headcount per dollar spent is obtained by targeting group 1 in comparison to groups 6, 5, 3 and 7. As for equations (10) and (4), group 1 shows the largest density around z_x. The second-best group to be targeted is group 2, whose unidimensional poverty impact per dollar spent is significantly larger for 3 and 7. A statistical ranking cannot be established with respect to the other geographical groups.

Let us now add the nutritional poverty component. This is shown in the second set of columns in Panel A of Table 4. When this term is added (the poverty headcount ratio in the second dimension), a significant re-ranking across the geographical groups is obtained. Groups 1 and 2 continue to be better prioritized by the targeting policy but comparisons with other groups have changed: group 1 is now preferred to groups 8 and 9 and no more to groups 3 and 6. Groups 3 and 6 show indeed the largest headcount poverty in nutrition — see Table 6. Taking multidimensional poverty into account then moves groups 3 and 6 upward in terms of priority, but not enough to outrank groups 1 and 2. Targeting group 2 is now statistically preferable compared to Targeting groups 9, 8 and 5. The next groups to be prioritized are groups 3, 10 and 6; targeting these groups provide statistically larger poverty reduction relative to targeting group 5.

For multidimensional poverty reduction, taking into account poverty in multiple dimensions is not enough. We must also take into account joint deprivation. This is done by adding the covariance term to the third set of columns in Table 6. A few changes in the ranking of priority groups are observable. Groups 1 and 2 are both the first groups to be prioritized; these groups are now statistically preferred to groups 5, 3, 8, 6, 4 and 7. Compared to the previous step, targeting group 1 now becomes statistically better than groups 3, 6 and 4, but loses its statistical priority on group 9; similarly, the poverty change in group 2 is now statistically larger than in groups 3, 6, 4 and 7, but not anymore with respect to group 9. Group 10 follows in the ranking; its poverty reduction is statistically larger than that in group 7. Finally, group 9 is preferred to groups 6 and 7 since, around the monetary poverty line, the covariance between its density and the poverty headcount in nutrition is relatively large.

The last set of columns shows the impact of adding spillover effects on priority rankings. This is done by adding the spill-over component, as indicated in equation (28). Groups 1 and 2 then lose their statistical priority over group 8. In addition, group 10 is now also preferred to group 6, while targeting group 5 allows for a statistically larger reduction in bi-dimensional population poverty in comparison to group 7.

A few interesting cases emerge comparing the policy guidance obtained under unidimensional poverty to that generated by a consideration of multidimensional poverty. As an example, from a uni-dimensional perspective (in both dimensions), there is no reason to prefer targeting group 1 relative to group 4. A preference for targeting group 1 instead of group 4 becomes, however, statistically significant under multidimensional poverty. Another case arises when comparing groups 9 and 7. When targeting monetary poverty, one cannot establish any statistical preference; when targeting nutritional poverty, prioritizing group 7 is statistically preferable to prioritizing group 9. Under a multidimensional approach, a statistical priority of group 9 over group 7 emerges.

Panel B of Table 4 shows priority rankings with $\alpha_x = \alpha_y = 1$. A priority re-ranking arises again when moving away from uni-dimensional towards multidimensional poverty alleviation. These higher values of α also yield greater ranking power. Under a unidimensional perspective (in both dimensions), group 5 does not emerge as a statistical priority over any other geographical groups. A preference for targeting group 5 compared to group 8 becomes, however, statistically significant from a multidimensional perspective; similarly, a preference for targeting group 7 as opposed to groups 4 and 6 cannot be established on the basis of unidimensional poverty, but it does become statistically significant under multidimensional poverty. On the contrary, targeting group 7 as opposed to group 1 can be rationalized under the objective of reducing unidimensional poverty, but not under the objective of alleviating multidimensional poverty.

As is well-known from the poverty literature, the use of different poverty indices can affect substantially the substantively conclusions obtained from poverty comparisons. As is less well known, that can also affect the comparative evaluation of the impact of targeting schemes, and thus the choice of policy design. This can be observed by comparing Panels A and B in Table 4. In particular, looking at the last set of columns (*Total impact with spill-over*), we see that targeting groups 3 and 7 is a statistically significant priority with $\alpha_x = \alpha_y = 1$; this is not the case with $\alpha_x = \alpha_y = 0$. Alternatively, there is no reason to prefer group 9 under the multidimensional poverty gap, while a statistically significant priority of group 9 over groups 6 and 7 arises with the multidimensional headcount.

More generally speaking, using the multidimensional poverty gap yields more precise targeting guidance. Greater statistical precision emerges because more statistical information is used with the poverty gap than with the headcount: when it comes to estimating standard errors, all observations below the poverty lines are important, not only those close to those lines. Greater normative strength is also obtained with the multidimensional poverty gap: the priority ranking with the multidimensional poverty gap is established by looking at the average welfare impact across all of the poor, and not only by considering whether that impact is large enough to lift some of the poor out of multidimensional poverty.

We now briefly present the results when the transfer is proportional (*i.e.* 1 percent of dimension x). The priority ranking can change substantially when we compare the results of a additive transfer with those obtained with a proportional transfer. This can be observed by comparing Tables 4 and 5. When $\alpha_x = \alpha_y = 0$, under the proportional case group 1 should also be preferred to groups 2, 8 and 9. These results are driven by the high average value of x in these groups (see Table 6). As we learned from equation (15), *ceteris paribus* for $\overline{x}$ larger than z_x (as in the case of groups 2, 8 and 9) the reduction in the bi-dimensional poverty index per dollar spent is slower than in the case where $\overline{x}$ is lower than z_x (as for group 1).

Table 4: Impact of targeting monetary dimension on bi-dimensional poverty: Vietnam 1992-1993 (*additive transfers*)

Ranking	Group	Population poverty change	Groups dominated	Group	Population poverty change	Groups dominated	Group	Population poverty change	Groups dominated	Group	Population poverty change	Groups dominated
***Panel A*: $\alpha_x = \alpha_y = 0$**												
	$-P(\alpha_x = -1)$			$-P(\alpha_x = -1)P(\alpha_y)$			$-[P(\alpha_x = -1)P(\alpha_y) + Cov(.)]$			Total impact with spill-over		
1	1	-0.000598	*6:5:3:7*	1	-0.000359	*7:9:8:5*	1	-0.000358	*5:3:8:6:4:7*	1	-0.000376	*5:3:6:7:4*
2	2	-0.000452	*3:7*	2	-0.000249	*9:8:5*	2	-0.000252	*5:3:8:6:4:7*	2	-0.000270	*5:3:6:7:4*
3	10	-0.000389	*10:1:2:9:5:8*	3	-0.000227	*5*	10	-0.000204	*7*	9	-0.000228	*6:7*
4	9	-0.000354	*10:2:9:5:8*	10	-0.000222	*5*	9	-0.000195	*6:7*	10	-0.000222	*6:7*
5	4	-0.000348	*9:5:8*	6	-0.000221	*5*	5	-0.000153		8	-0.000200	
6	6	-0.000327	*9:5:8*	4	-0.000216		3	-0.000132		5	-0.000184	*7*
7	5	-0.000324	*9:5:8*	7	-0.000194		8	-0.000126		3	-0.000136	
8	3	-0.000320		9	-0.000159		6	-0.000105		6	-0.000116	
9	8	-0.000314		8	-0.000135		4	-0.000103		7	-0.000108	
10	7	-0.000283		5	-0.000128		7	-0.000100		4	-0.000106	
***Panel B*: $\alpha_x = \alpha_y = 1$**												
	$-(\alpha_x/z_x)[P(\alpha_x - 1)]$			$-(\alpha_x/z_x)[P(\alpha_x - 1)P(\alpha_y)]$			$-(\alpha_x/z_x)[P(\alpha_x - 1)P(\alpha_y) + Cov(.)]$			Total impact with spill-over		
1	4	-0.000520	*7:6:10:1:2:9:5:8*	3	-0.000050	*6:4:10:2:9:5:8*	3	-0.000052	*6:4:10:2:5:9:8*	3	-0.000057	*6:4:10:2:5:9:8*
2	3	-0.000499	*6:10:1:2:9:5:8*	7	-0.000048	*6:4:10:2:9:5:8*	7	-0.000050	*6:4:10:2:5:9:8*	7	-0.000055	*6:4:10:2:5:9:8*
3	7	-0.000481	*10:1:2:9:5:8*	6	-0.000039	*10:2:9:5:8*	1	-0.000043	*2:5:9:8*	1	-0.000047	*5:9:8*
4	6	-0.000463	*10:2:9:5:8*	1	-0.000037	*9:5:8*	6	-0.000040	*2:5:9:8*	6	-0.000044	*2:5:9:8*
5	10	-0.000394	*9:5:8*	4	-0.000037	*2:9:5:8*	4	-0.000037	*5:9:8*	4	-0.000041	*5:9:8*
6	1	-0.000388	*9:5:8*	10	-0.000029	*9:5:8*	10	-0.000033	*5:9:8*	10	-0.000036	*5:9:8*
7	2	-0.000374	*9:5:8*	2	-0.000027	*9:5:8*	2	-0.000029	*5:9:8*	2	-0.000032	*5:9:8*
8	9	-0.000226		9	-0.000009	*8*	5	-0.000011	*8*	5	-0.000012	*8*
9	5	-0.000170		5	-0.000007		9	-0.000007		9	-0.000008	
10	8	-0.000116		8	-0.000004		8	-0.000004		8	-0.000004	

Source: authors' analysis based on data from the VLSS 1992-1993.

Note: The "groups dominated" (in *italics*) are those groups for which the difference in poverty impact is significant at 5 percent.

Table 5: Impact of targeting monetary dimension on bi-dimensional poverty: Vietnam 1992-1993 (*proportional transfers*)

Ranking	Group	Population poverty change	Groups dominated	Group	Population poverty change	Groups dominated	Group	Population poverty change	Groups dominated	Group	Population poverty change	Groups dominated
Panel A: $\alpha_x = \alpha_y = 0$												
	$-(z_x/\overline{x})P(\alpha_x = -1)$			$-(z_x/\overline{x})P(\alpha_x = -1)P(\alpha_y)$			$-(z_x/\overline{x})[P(\alpha_x = -1)P(\alpha_y) + Cov(.)]$			Total impact with spill-over		
1	1	-0.0000068	*6:7:9:5:8*	1	-0.0000041	*2:9:5:8*	1	-0.0000041	*2:3:4:6:7:9:5:8*	1	-0.0000041	*2:3:4:6:7:9:5:8*
2	4	-0.0000052	*5:8*	3	-0.0000032	*9:5:8*	2	-0.0000025	*6:7:9:5:8*	2	-0.0000025	*6:7:9:5:8*
3	3	-0.0000045	*9:5:8*	4	-0.0000032	*9:5:8*	10	-0.0000024	*5:8*	10	-0.0000024	*5:8*
4	10	-0.0000045	*9:5:8*	6	-0.0000029	*9:5:8*	3	-0.0000019	*8*	3	-0.0000019	*8*
5	2	-0.0000044	*9:5:8*	7	-0.0000026	*9:5:8*	4	-0.0000015		4	-0.0000015	
6	6	-0.0000042	*9:5:8*	10	-0.0000026	*9:5:8*	6	-0.0000014		6	-0.0000014	
7	7	-0.0000038	*5:8*	2	-0.0000024	*9:5:8*	7	-0.0000013		7	-0.0000013	
8	9	-0.0000023		9	-0.0000011		9	-0.0000013		9	-0.0000013	
9	5	-0.0000020		5	-0.0000008		5	-0.0000009		5	-0.0000009	
10	8	-0.0000017		8	-0.0000007		8	-0.0000007		8	-0.0000007	
Panel B: $\alpha_x = \alpha_y = 1$												
	$-(\alpha_x/\overline{x})[P(\alpha_x - 1) - P(\alpha_x)]$			$-(\alpha_x/\overline{x})[(P(\alpha_x - 1) - P(\alpha_x))P(\alpha_y)]$			$-(\alpha_x/\overline{x})[(P(\alpha_x - 1) - P(\alpha_x))P(\alpha_y) + Cov(.)]$			Total impact with spill-over		
1	4	-0.0000049	*7:6:10:1:2:9:5:8*	3	-0.0000005	*4:6:1:10:2:9:5:8*	3	-0.0000005	*4:6:1:10:2:5:9:8*	3	-0.0000005	*4:6:1:10:2:9:8*
2	3	-0.0000045	*6:10:1:2:9:5:8*	7	-0.0000004	*6:1:10:2:9:5:8*	7	-0.0000004	*6:1:10:2:5:9:8*	7	-0.0000005	*6:1:10:2:9:8*
3	7	-0.0000042	*10:1:2:9:5:8*	4	-0.0000004	*10:2:9:5:8*	4	-0.0000004	*10:2:5:9:8*	4	-0.0000004	*10:2:9:8*
4	6	-0.0000040	*10:1:2:9:5:8*	6	-0.0000003	*10:2:9:5:8*	6	-0.0000003	*10:2:5:9:8*	6	-0.0000004	*10:2:9:8*
5	10	-0.0000029	*9:5:8*	1	-0.0000003	*9:5:8*	1	-0.0000003	*5:9:8*	1	-0.0000003	*9:8*
6	1	-0.0000028	*9:5:8*	10	-0.0000002	*9:5:8*	10	-0.0000002	*5:9:8*	10	-0.0000003	*9:8*
7	2	-0.0000025	*9:5:8*	2	-0.0000002	*9:5:8*	2	-0.0000002	*5:9:8*	2	-0.0000002	*9:8*
8	9	-0.0000011		9	-0.0000000		5	-0.0000001		5	-0.0000001	
9	5	-0.0000008		5	-0.0000000		9	-0.0000000		9	-0.0000000	
10	8	-0.0000005		8	-0.0000000		8	-0.0000000		8	-0.0000000	

Source: authors' analysis based on data from the VLSS 1992-1993.

Note: The "groups dominated" (in *italics*) are those groups for which the difference in poverty impact is significant at 5 percent.

Table 6: Population shares and poverty gaps in the monetary and nutritional dimensions: Vietnam 1992-1993

Groups	**Population shares**	**Monetary**			**Nutrition**		
		P0	P1	mean	P0	P1	mean
1	0.034	0.695	0.249	1565.785	0.600	0.096	7.617
2	0.260	0.669	0.211	1834.894	0.551	0.073	7.958
3	0.160	0.893	0.321	1262.547	0.710	0.100	7.459
4	0.036	0.930	0.336	1209.299	0.620	0.071	7.711
5	0.070	0.304	0.077	2936.803	0.397	0.041	8.414
6	0.172	0.829	0.277	1387.726	0.678	0.083	7.698
7	0.145	0.862	0.307	1329.38	0.686	0.101	7.521
8	0.021	0.207	0.031	3348.073	0.431	0.031	8.239
9	0.032	0.404	0.119	2699.172	0.449	0.038	8.431
10	0.068	0.706	0.261	1552.488	0.572	0.074	7.884
Population	1	0.729	0.247	1679.308	0.607	0.080	7.798

Source: authors' analysis based on data from the VLSS 1992-1993

Moving to Vietnam 1997-1998, a similarly large re-ranking across groups is observed when shifting from unidimensional to multidimensional poverty under both with $\alpha_x = \alpha_y = 0$ and $\alpha_x = \alpha_y = 1$. The results for the additive transfer are shown in Table 7, panels A and B, and in Table 9. A notable difference with previous results concerns the spillover effect of dimensional targeting. With $\alpha_x = \alpha_y = 0$, moving from deprivation in each of the two dimensions to complete multidimensional poverty (*i.e.*, taking into account the spill-over effect across dimensions) does not statistically affect the priority ranking of group targeting. For $\alpha_x = \alpha_y = 0$, the spillover effect is marginal.

As for Vietnam 1992-1993, when we move to the proportional case (see Table 8), the priority ranking across groups changes in comparison with the additive case. As an example, with $\alpha_x = \alpha_y = 1$ (Panel B), under the proportional transfer group 8 cannot be preferred anymore to groups 4 and 5. As stated in equation 14, wider the difference between $P(\alpha_x - 1)$ and $P(\alpha_x)$ faster is the reduction in the multidimensional poverty index. As shown in Table 9, these two groups have the lowest $P(\alpha_x = 1)$; also, the distance between $P(\alpha_x = 0)$ and $P(\alpha_x = 1)$ is large enough that targeting group 8 should not be prioritized in comparison with groups 4 and 5.

Table 7: Impact of targeting monetary dimension on bi-dimensional poverty: Vietnam 1997-1998 (*additive transfers*)

Ranking	Group	Population poverty change	Groups dominated	Group	Population poverty change	Groups dominated	Group	Population poverty change	Groups dominated	Group	Population poverty change	Groups dominated
Panel A: $\alpha_x = \alpha_y = 0$												
	$-P(\alpha_x = -1)$			$-P(\alpha_x = -1)P(\alpha_y)$			$-[P(\alpha_x = -1)P(\alpha_y) + Cov(.)]$			Total impact with spill-over		
1	1	-0.000633	*6:8:7:5:4*	1	-0.000283	*6:8:7:5:4*	1	-0.000313	*9:7:5:8:4*	1	-0.000337	*9:7:5:8:4*
2	2	-0.000568	*6:8:7:5:4*	10	-0.000283	*6:8:7:5:4*	2	-0.000258	*5:8:4*	2	-0.000274	*5:8:4*
3	3	-0.000529	*4*	2	-0.000274	*6:8:7:5:4*	10	-0.000230	*8:4*	10	-0.000243	*8:4*
4	10	-0.000518	*6:7:5:4*	9	-0.000238	*7:5:4*	6	-0.000197	*4*	6	-0.000222	*4*
5	9	-0.000426	*5:4*	3	-0.000233	*5:4*	9	-0.000164		9	-0.000176	
6	6	-0.000330	*5:4*	6	-0.000112	*5:4*	3	-0.000158		3	-0.000172	
7	8	-0.000289		8	-0.000105		7	-0.000145		7	-0.000168	
8	7	-0.000262	*4*	7	-0.000092	*5:4*	5	-0.000139		5	-0.000163	
9	5	-0.000188		5	-0.000038		8	-0.000120		8	-0.000148	
10	4	-0.000107		4	-0.000021		4	-0.000065		4	-0.000085	
Panel B: $\alpha_x = \alpha_y = 1$												
	$-(\alpha_x/z_x)[P(\alpha_x - 1)]$			$-(\alpha_x/z_x)[P(\alpha_x - 1)P(\alpha_y)]$			$-(\alpha_x/z_x)[P(\alpha_x - 1)P(\alpha_y) + Cov(.)]$			Total impact with spill-over		
1	3	-0.000432	*1:2:6:8:7:5:4*	9	-0.000025	*1:6:8:7:4:5*	9	-0.000028	*3:1:8:6:7:4:5*	9	-0.000030	*3:1:8:6:7:4:5*
2	10	-0.000363	*1:2:6:8:7:5:4*	10	-0.000021	*1:6:8:7:4:5*	10	-0.000023	*1:8:6:7:4:5*	10	-0.000024	*1:8:6:7:4:5*
3	9	-0.000350	*6:8:7:5:4*	2	-0.000015	*6:8:7:4:5*	2	-0.000018	*6:7:4:5*	2	-0.000018	*8:6:7:4:5*
4	1	-0.000277	*6:8:7:5:4*	3	-0.000012	*4:5*	3	-0.000013	*4:5*	3	-0.000015	*4:5*
5	2	-0.000277	*6:8:7:5:4*	1	-0.000011	*6:4:5*	1	-0.000012	*4:5*	1	-0.000012	*4:5*
6	6	-0.000161	*5:4*	6	-0.000005	*4:5*	8	-0.000010	*4:5*	8	-0.000010	*4:5*
7	8	-0.000141		8	-0.000005		6	-0.000009	*4:5*	6	-0.000009	*4:5*
8	7	-0.000110		7	-0.000004		7	-0.000007	*5*	7	-0.000007	*5*
9	5	-0.000065		4	-0.000001		4	-0.000002		4	-0.000002	
10	4	-0.000058		5	-0.000001		5	-0.000001		5	-0.000001	

Source: authors' analysis based on data from the VLSS 1997-1998.

Note: The "groups dominated" (in *italics*) are those groups for which the difference in poverty impact is significant at 5 percent.

Table 8: Impact of targeting monetary dimension on bi-dimensional poverty: Vietnam 1997-1998 (*proportional transfers*)

Ranking	Group	Population poverty change	Groups dominated	Group	Population poverty change	Groups dominated	Group	Population poverty change	Groups dominated	Group	Population poverty change	Groups dominated
Panel A: $\alpha_x = \alpha_y = 0$												
	$-(z_x/\overline{x})P(\alpha_x = -1)$			$-(z_x/\overline{x})P(\alpha_x = -1)P(\alpha_y)$			$-(z_x/\overline{x})[P(\alpha_x = -1)P(\alpha_y) + Cov(.)]$			Total impact with spill-over		
1	3	-0.0000066	*6:8:7:5:4*	3	-0.0000029	*6:8:7:5:4*	1	-0.0000027	*6:7:8:5:4*	1	-0.0000027	*6:7:8:5:4*
2	1	-0.0000055	*6:8:7:5:4*	10	-0.0000029	*6:8:7:5:4*	10	-0.0000023	*6:7:8:5:4*	10	-0.0000023	*6:7:8:5:4*
3	10	-0.0000053	*6:8:7:5:4*	2	-0.0000025	*6:8:7:5:4*	2	-0.0000023	*6:7:8:5:4*	2	-0.0000023	*6:7:8:5:4*
4	2	-0.0000051	*6:8:7:5:4*	9	-0.0000025	*6:8:7:5:4*	3	-0.0000020		3	-0.0000020	
5	9	-0.0000044	*7:5:4*	1	-0.0000025	*6:8:7:5:4*	9	-0.0000017	*4*	9	-0.0000017	*4*
6	6	-0.0000022	*5:4*	6	-0.0000007	*5:4*	6	-0.0000013	*5:4*	6	-0.0000013	*5:4*
7	8	-0.0000015		8	-0.0000006		7	-0.0000008		7	-0.0000008	
8	7	-0.0000014	*4*	7	-0.0000005	*5:4*	8	-0.0000006		8	-0.0000006	
9	5	-0.0000007		5	-0.0000001		5	-0.0000005		5	-0.0000005	
10	4	-0.0000003		4	-0.0000001		4	-0.0000002		4	-0.0000002	
Panel B: $\alpha_x = \alpha_y = 1$												
	$-(\alpha_x/\overline{x})[P(\alpha_x - 1) - P(\alpha_x)]$			$-(\alpha_x/\overline{x})[(P(\alpha_x - 1) - P(\alpha_x))P(\alpha_y)]$			$-(\alpha_x/\overline{x})[(P(\alpha_x - 1) - P(\alpha_x))P(\alpha_y) + Cov(.)]$			Total impact with spill-over		
1	3	-0.0000036	*2:1:6:8:7:5:4*	9	-0.0000002	*6:8:7:5:4*	9	-0.0000002	*8:6:7:4:5*	9	-0.0000002	*8:6:7:4:5*
2	10	-0.0000026	*2:1:6:8:7:5:4*	10	-0.0000001	*2:1:6:8:7:5:4*	10	-0.0000002	*1:8:6:7:4:5*	10	-0.0000002	*1:8:6:7:4:5*
3	9	-0.0000023	*6:8:7:5:4*	3	-0.0000001	*7:5:4*	2	-0.0000001	*8:6:7:4:5*	2	-0.0000001	*6:7:4:5*
4	2	-0.0000019	*6:8:7:5:4*	2	-0.0000001	*6:8:7:5:4*	3	-0.0000001	*7:4:5*	3	-0.0000001	*7:4:5*
5	1	-0.0000019	*6:8:7:5:4*	1	-0.0000001	*7:5:4*	1	-0.0000001	*7:4:5*	1	-0.0000001	*7:4:5*
6	6	-0.0000007	*5:4*	6	-0.0000000		8	-0.0000000		8	-0.0000000	
7	8	-0.0000006		8	-0.0000000		6	-0.0000000	*4:5*	6	-0.0000000	*4:5*
8	7	-0.0000004		7	-0.0000000		7	-0.0000000		7	-0.0000000	
9	5	-0.0000002		5	-0.0000000		4	-0.0000000		4	-0.0000000	
10	4	-0.0000001		4	-0.0000000		5	-0.0000000		5	-0.0000000	

Source: authors' analysis based on data from the VLSS 1997-1998.

Note: The "groups dominated" (in *italics*) are those groups for which the difference in poverty impact is significant at 5 percent.

Table 9: Population shares and poverty gaps in the monetary and nutritional dimensions: Vietnam 1997-1998

Groups	**Population shares**	**Monetary**			**Nutrition**		
		P0	P1	mean	P0	P1	mean
1	0.123	0.495	0.113	2064.115	0.448	0.039	8.299
2	0.223	0.495	0.122	1980.894	0.481	0.053	8.228
3	0.028	0.773	0.255	1427.64	0.440	0.028	8.483
4	0.059	0.105	0.022	5605.957	0.198	0.018	8.98
5	0.031	0.116	0.015	4835.542	0.202	0.013	9.166
6	0.103	0.288	0.087	2740.731	0.339	0.032	8.579
7	0.018	0.197	0.063	3278.567	0.350	0.034	8.41
8	0.030	0.252	0.048	3350.969	0.363	0.036	8.449
9	0.049	0.627	0.232	1738.273	0.560	0.073	7.922
10	0.337	0.649	0.188	1755.917	0.546	0.057	8.048
Population	1	0.493	0.136	2332.586	0.456	0.047	8.287

Source: authors' analysis based on data from the VLSS 1997-1998

Let us now turn to regional targeting in South Africa. The main results for the additive case are shown in Table 10. Let us concentrate on the results with $\alpha_x = \alpha_y = 1$ (Panel B) and on some of the more interesting findings. Take group 17, for instance; it has the largest health headcount among almost all geographical groups (see Table 12) as well as an above average health poverty gap, but its level of monetary poverty is not statistically larger than any of the other groups. With multidimensional poverty, a statistically significant preference for targeting group 17 can be established only with respect to group 7. Targeting group 11 is better than targeting group 12 in terms of unidimensional poverty in both the monetary and the health dimensions, but this is nevertheless not the case when the effect of such targeting on multidimensional poverty is taken into account.

When comparing results for total multidimensional poverty with $\alpha_x = \alpha_y = 0$ and $\alpha_x = \alpha_y = 1$, the policy guidance changes dramatically. This is easily seen by comparing panels A and B of Table 10. As an example, group 3 is dominated by most of other geographical groups when $\alpha_x = \alpha_y = 0$, while, with $\alpha_x = \alpha_y = 1$, it dominates 16 out of 17 possible groups (group 5 is the only group not statistically outranked by group 3). While group 13 shows an extraordinarily large monetary headcount and nutritional poverty gap (which explains its high priority ranking under $\alpha = 1$: Figure 1, panel A), nearly nobody lies around the monetary poverty line (which explains the small bi-dimensional impact when $\alpha = 0$: panel B). This important distinction between the incidence and the intensity of multidimensional poverty, and between levels of multidimensional poverty and strategies for efficient multidimensional poverty alleviation, explains the important reversals of priority rankings when moving from $\alpha_x = \alpha_y = 0$ to $\alpha_x = \alpha_y = 1$.

Finally, the priority ranking can change substantially when the proportional transfer approach is adopted (see Table 11). As an example, with $\alpha_x = \alpha_y = 0$ (Panel A), under a proportional transfer group 13 should be preferred to many groups, while under the additive approach it should be prioritized only in comparison with group 3. Indeed, as shown in Table , group 13 has one of the lowest $\overline{x}$ which is well below the poverty line.

Table 10: Impact of monetary targeting on bi-dimensional poverty: South Africa 1993 (*additive transfers*)

Ranking	Group	Population poverty change	Groups dominated	Group	Population poverty change	Groups dominated	Group	Population poverty change	Groups dominated	Group	Population poverty change	Groups dominated
Panel A: $\alpha_x = \alpha_y = 0$												
	$-P(\alpha_x = -1)$			$-P(\alpha_x = -1)P(\alpha_y)$			$-[P(\alpha_x = -1)P(\alpha_y) + Cov(.)]$			Total impact with spill-over		
1	4	-0.00458	*18:3*	17	-0.00133	*2:12:16:1:3:14:18:7*	4	-0.00307	*3*	4	-0.00315	*3*
2	6	-0.00385	*12:1:7:2:16:18:3*	5	-0.00125	*18:7*	15	-0.00295	*13:1:17:18:16:2:3*	15	-0.00308	*13:1:17:18:16:2:3*
3	8	-0.00383	*1:2:18:3*	6	-0.00125	*2:12:16:1:3:14:18:7*	6	-0.00253	*1:17:18:16:2:3*	6	-0.00268	*1:17:18:16:2:3*
4	15	-0.00365	*1:7:2:16:18:3*	4	-0.00119		9	-0.00221		9	-0.00233	
5	5	-0.00348	*1:2:18:3*	8	-0.00109	*7*	8	-0.00221	*18:3*	8	-0.00231	*3*
6	17	-0.00323	*18:3*	15	-0.00105	*12:1:3:14:18:7*	10	-0.00208	*3*	12	-0.00226	*3*
7	10	-0.00319	*18:3*	13	-0.00100	*2:12:1:3:14:18:7*	11	-0.00207	*18:3*	10	-0.00225	*3*
8	11	-0.00304	*18:3*	10	-0.00098	*12:1:3:14:18:7*	5	-0.00201	*3*	11	-0.00224	*3*
9	13	-0.00288	*18:3*	11	-0.00085	*1:3:18:7*	12	-0.00200	*3*	5	-0.00215	*3*
10	9	-0.00259		9	-0.00073		13	-0.00193	*3*	14	-0.00210	*3*
11	12	-0.00258	*3*	2	-0.00067	*18:7*	14	-0.00193		13	-0.00203	*3*
12	14	-0.00235		12	-0.00050		7	-0.00184	*3*	7	-0.00203	*3*
13	1	-0.00225	*3*	16	-0.00050		1	-0.00154	*3*	1	-0.00179	*3*
14	7	-0.00215		1	-0.00047		17	-0.00143	*3*	17	-0.00178	*3*
15	2	-0.00189		3	-0.00045		18	-0.00135	*3*	18	-0.00159	*3*
Panel B: $\alpha_x = \alpha_y = 1$												
	$-(\alpha_x/z_x)[P(\alpha_x - 1)]$			$-(\alpha_x/z_x)[P(\alpha_x - 1)P(\alpha_y)]$			$-(\alpha_x/z_x)[P(\alpha_x - 1)P(\alpha_y) + Cov(.)]$			Total impact with spill-over		
1	3	-0.00544	*13:6:15:11:4:10:2:5: 12:16:17:14:8:7:1:18*	3	-0.00026	*13:6:4:2:15:10: 9:11:8:17:16:12: 1:18:7:14*	3	-0.00027	*13:6:10:15:4:8: 2:9:11:16:12:17: 18:14:1:7*	3	-0.00031	*13:6:10:15:9:4: 8:2:11:16:12:17: 14:18:1:7*
2	9	-0.00478	*11:2:5:12:16:17:14:8: 7:1:18*	13	-0.00018	*9:11:8:17:16:12: 1:18:7:14*	13	-0.00020	*9:11:12:17:18:14: 1:7*	13	-0.00024	*9:2:11:12:17:14: 7*
3	13	-0.00460	*6:15:11:2:5:12:16: 17:14:8:7:1:18*	6	-0.00016	*11:8:17:16:12:1: 18:7:14*	5	-0.00020		5	-0.00022	
4	6	-0.00396	*5:12:16:17:14:8: 7:1:18*	5	-0.00013		6	-0.00017	*12:17:18:14:1:7*	6	-0.00018	*12:17:14:18:1:7*
5	15	-0.00384	*12:16:17:14:8:7:1:18*	4	-0.00013	*12:1:18:7:14*	10	-0.00016	*12:17:18:14:1:7*	10	-0.00018	*12:17:14:18:1:7*
6	11	-0.00371	*12:17:14:8:7:1:18*	2	-0.00012	*12:1:18:7:14*	15	-0.00015	*12:17:18:14:1:7*	15	-0.00018	*12:17:14:18:1:7*
7	4	-0.00361	*12:17:14:8:7:1:18*	15	-0.00012	*12:1:18:7:14*	4	-0.00014	*17:18:14:1:7*	9	-0.00016	*12:17:14:18:1:7*
8	10	-0.00360	*12:17:14:8:7:1:18*	10	-0.00012	*12:1:18:7:14*	8	-0.00014	*7*	4	-0.00015	*14:18:1:7*
9	2	-0.00314	*17:8:7:1:18*	9	-0.00011	*12:1:18:7:14*	2	-0.00013	*18:14:1:7*	8	-0.00015	*18:1:14*
10	5	-0.00233		11	-0.00010	*12:1:18:7:14*	9	-0.00013	*17:18:14:1:7*	2	-0.00015	*18:1:14*
11	12	-0.00203	*18*	8	-0.00008		11	-0.00012	*17:18:14:1:7*	11	-0.00014	*17:14:18:1:7*
12	16	-0.00194		17	-0.00007	*18:7:14*	16	-0.00010		16	-0.00011	
13	17	-0.00179		16	-0.00005		12	-0.00007	*7*	12	-0.00008	*7*
14	14	-0.00174		12	-0.00004		17	-0.00006	*7*	17	-0.00008	*7*
15	8	-0.00167		1	-0.00003		18	-0.00005	*7*	14	-0.00006	
16	7	-0.00165		18	-0.00002		14	-0.00005		18	-0.00005	*7*

Source: authors' analysis based on data from the SAIHS 1993.

Notes: The "groups dominated" (in *italics*) are those groups for which the difference in poverty impact is significant at 5 percent;
Lower ranked groups that do not dominate any group are not reported in the table for lack of space.

Table 11: Impact of monetary targeting on bi-dimensional poverty: South Africa 1993 (*proportional transfers*)

Ranking	Group	Population poverty change	Groups dominated	Group	Population poverty change	Groups dominated	Group	Population poverty change	Groups dominated	Group	Population poverty change	Groups dominated
***Panel A**:* $\alpha_x = \alpha_y = 0$												
	$-(z_x/\overline{x})P(\alpha_x = -1)$			$-(z_x/\overline{x})P(\alpha_x = -1)P(\alpha_y)$			$-(z_x/\overline{x})[P(\alpha_x = -1)P(\alpha_y) + Cov(.)]$			Total impact with spill-over		
1	6	-0.000040	*11:10:8:17:3:2:12:14:7:16:1:18*	6	-0.000013	*11:3:10:2:16:12:14:1:7:18*	9	-0.000028		9	-0.000028	
2	13	-0.000034	*17:3:2:12:14:7:16:1:18*	13	-0.000012	*11:3:2:16:12:14:1:7:18*	6	-0.000026	*5:10:8:3:2:17:14:7:12:16:1:18*	6	-0.000026	*5:10:8:3:2:17:14:7:12:16:1:18*
3	9	-0.000033		9	-0.000009		15	-0.000024	*8:3:2:17:14:7:12:16:1:18*	15	-0.000024	*8:3:2:17:14:7:12:16:1:18*
4	15	-0.000030	*12:14:7:16:1:18*	5	-0.000009		13	-0.000023	*8:3:2:17:14:7:12:16:1:18*	13	-0.000023	*8:3:2:17:14:7:12:16:1:18*
5	4	-0.000029		17	-0.000009	*16:12:14:1:7:18*	4	-0.000020		4	-0.000020	
6	11	-0.000026	*12:14:7:16:1:18*	15	-0.000009	*16:12:14:1:7:18*	11	-0.000018	*17:7:12:16:1:18*	11	-0.000018	*17:7:12:16:1:18*
7	5	-0.000025	*7:1:18*	4	-0.000008		5	-0.000015	*18*	5	-0.000015	*18*
8	10	-0.000022	*1:18*	11	-0.000007	*12:14:1:7:18*	10	-0.000014	*18*	10	-0.000014	*18*
9	8	-0.000021	*18*	3	-0.000007	*12:14:1:7:18*	8	-0.000012	*18*	8	-0.000012	*18*
10	17	-0.000021	*7:1:18*	10	-0.000007	*12:1:7:18*	3	-0.000011	*1:18*	3	-0.000011	*1:18*
11	3	-0.000018	*1:18*	8	-0.000006		2	-0.000011		2	-0.000011	
12	2	-0.000016	*18*	2	-0.000006	*12:14:1:7:18*	17	-0.000009	*18*	17	-0.000009	*18*
***Panel B**:* $\alpha_x = \alpha_y = 1$												
	$-(\alpha_x/\overline{x})[P(\alpha_x - 1) - P(\alpha_x)]$			$-(\alpha_x/\overline{x})[(P(\alpha_x - 1) - P(\alpha_x))P(\alpha_y)]$			$-(\alpha_x/\overline{x})[(P(\alpha_x - 1) - P(\alpha_x))P(\alpha_y) + Cov(.)]$			Total impact with spill-over		
1	3	-0.000039	*13:6:11:15:2:4:10:5:17:8:12:16:7:14:1:18*	3	-0.000002	*13:6:9:5:2:15:4:11:10:8:17:16:12:1:7:18*	3	-0.000002	*6:13:9:2:11:4:15:10:8:17:16:12:1:14:18:7*	3	-0.000002	*13:6:9:5:2:15:11:10:4:8:17:16:12:14:1:18:7*
2	9	-0.000030	*11:15:2:4:10:5:17:8:12:16:7:14:1:18*	13	-0.000001	*9:2:15:11:10:8:17:16:12:1:7:18*	6	-0.000001	*11:15:10:8:17:16:12:1:14:18:7*	13	-0.000001	*2:15:11:10:4:8:17:16:12:14:1:18:7*
3	13	-0.000029	*11:15:2:4:10:5:17:8:12:16:7:14:1:18*	6	-0.000001	*15:11:10:8:17:16:12:1:7:18*	13	-0.000001	*11:15:10:8:17:16:12:1:14:18:7*	6	-0.000001	*11:8:17:16:12:14:1:18:7*
4	6	-0.000026	*11:15:2:10:5:17:8:12:16:7:14:1:18*	9	-0.000001	*8:17:16:12:1:7:18*	5	-0.000001		9	-0.000001	*17:16:12:14:1:18:7*
5	11	-0.000019	*17:8:12:16:7:14:1:18*	5	-0.000001		9	-0.000001	*17:16:12:1:14:18:7*	5	-0.000001	
6	15	-0.000018	*17:8:12:16:7:14:1:18*	2	-0.000001	*12:1:7:18*	2	-0.000001	*1:14:18:7*	2	-0.000001	*12:14:1:18:7*
7	2	-0.000016	*8:12:7:14:1:18*	15	-0.000001	*12:1:7:18*	11	-0.000001	*17:12:1:14:18:7*	15	-0.000001	*17:12:14:1:18:7*
8	4	-0.000015	*18*	4	-0.000001		4	-0.000001	*18:7*	11	-0.000001	*17:12:14:1:18:7*
9	10	-0.000014	*1:18*	11	-0.000001	*12:1:7:18*	15	-0.000001	*17:12:1:14:18:7*	10	-0.000001	*14:1:18:7*
10	5	-0.000011		10	0.000000		10	-0.000001	*1:14:18:7*	4	-0.000001	*18:7*
11	17	-0.000007	*18*	8	0.000000		8	-0.000001		8	-0.000001	
12	8	-0.000006		17	0.000000	*1:7:18*	17	0.000000	*18*	17	0.000000	*1:18:7*

Source: authors' analysis based on data from the SAIHS 1993.
Notes: The "groups dominated" (in *italics*) are those groups for which the difference in poverty impact is significant at 5 percent;
Lower ranked groups that do not dominate any group are not reported in the table for lack of space.

Table 12: Population shares and poverty gaps in the monetary and nutritional dimensions: South Africa 1993

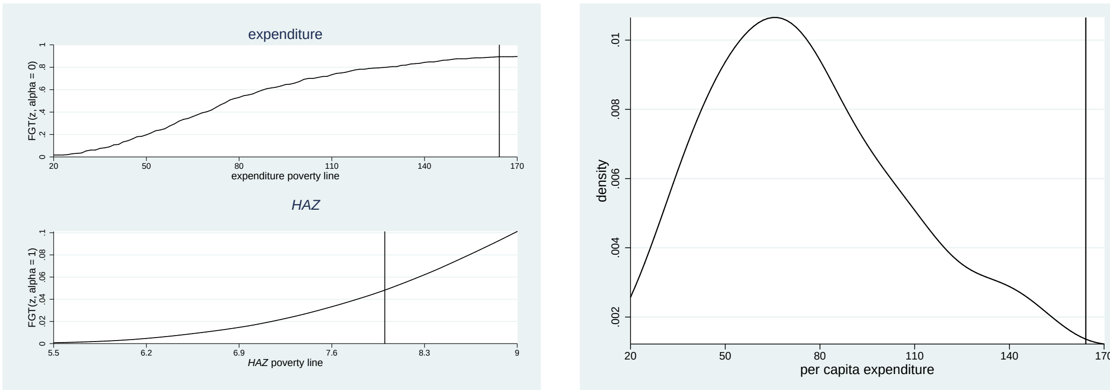
Groups	Population shares	Monetary			Nutrition		
		P0	P1	mean	P0	P1	mean
1	0.068	0.251	0.078	473.337	0.210	0.021	9.109
2	0.013	0.515	0.207	194.412	0.356	0.040	8.356
3	0.146	0.894	0.473	107.529	0.384	0.048	8.474
4	0.016	0.593	0.209	256.718	0.261	0.036	8.993
5	0.015	0.383	0.125	226.900	0.361	0.057	8.404
6	0.140	0.649	0.232	158.029	0.324	0.041	8.646
7	0.038	0.271	0.085	365.650	0.115	0.014	9.211
8	0.022	0.274	0.097	294.640	0.286	0.046	8.903
9	0.024	0.785	0.399	130.403	0.284	0.024	8.901
10	0.034	0.591	0.250	238.046	0.306	0.033	8.740
11	0.063	0.610	0.252	189.990	0.279	0.028	8.772
12	0.025	0.333	0.096	402.422	0.196	0.018	8.781
13	0.151	0.755	0.355	138.297	0.346	0.039	8.517
14	0.011	0.285	0.118	376.269	0.171	0.012	9.221
15	0.057	0.631	0.274	199.732	0.287	0.031	8.727
16	0.012	0.318	0.125	354.744	0.273	0.026	9.359
17	0.029	0.294	0.123	252.045	0.412	0.038	8.375
18	0.137	0.196	0.063	600.252	0.193	0.021	9.301
National	1	0.554	0.241	263.750	0.292	0.034	8.781

Source: authors' analysis based on data from the SAIHS 1993

The results above showed how a progressive switch from unidimensional to multidimensional poverty changes the targeting strategies that must be efficiently chosen for reducing poverty. They also showed that priority rankings can sometimes demand on how multidimensional poverty is measured. Section 2.1 discussed, however, how we can use our bi-dimensional headcount indices over areas of bi-dimensional poverty frontiers to build multidimensional dominance surfaces and to assess whether priority rankings for group targeting are robust over classes of multidimensional poverty indices.

This section illustrates this by presenting results of univariate and bivariate dominance tests. Recall that the condition for a distribution *A* to dominate a distribution *B* is that A's dominance surface be lower than that of *B* over a sufficiently large area of poverty frontiers. In terms of targeting dominance, the same result applies but this time by comparing the targeting surface of a group A to that of a group B. Prioritizing a group with a more

Figure 1: Expenditure density and FGT indices for South Africa's group 3



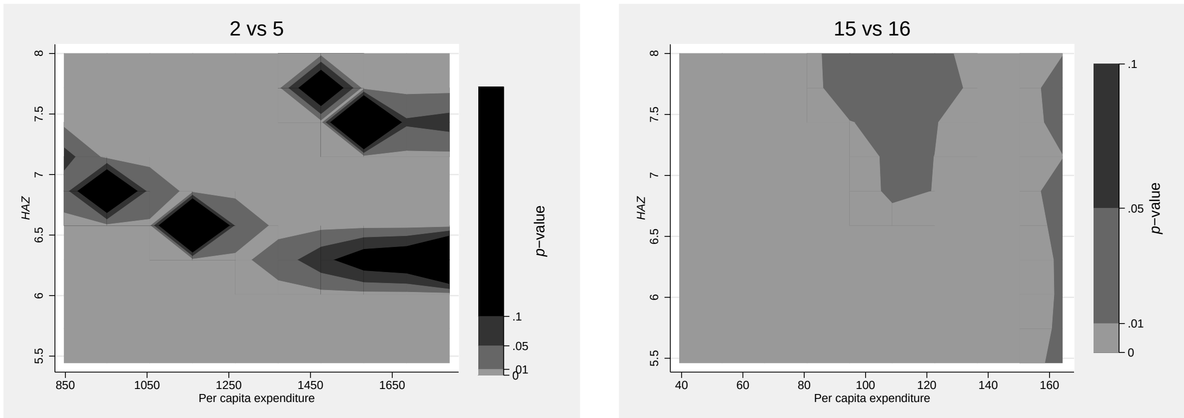
(a) FGT indices (b) Expenditure density

Source: authors' analysis based on data from SAIHS 1993

negative targeting surface will lead to a faster reduction in multidimensional poverty per dollar spent the

Let us start with multidimensional poverty dominance tests. We specify 10 different poverty lines for each of the two dimensions, giving an area of poverty frontiers set over 100 possible combinations of monetary and health poverty lines. The 10 poverty lines in each dimension are set at the the minimum values of the indicators plus the deciles of the distance between the official poverty lines and those minimum values. The upper limit of those lines (the upper right corners in the forthcoming figures) corresponds to the official poverty lines, while the lower poverty lines are at the lower left corners.

Figure 2: Testing targeting dominance for the class of $\Pi^{1,1}$ indices (*additive transfers*)



(a) Vietnam 1992-1993 (b) South Africa 1993

Note: the first graph shows the p-values of the differences in poverty impact between targeting group 2 and targeting group 5 in Vietnam; the second graph shows the p-values of the differences in poverty impact between targeting group 2 and targeting group 5 in South Africa. Lighter areas indicate where it is statistically more likely that targeting the first group (in the two graphs respectively) will reduce poverty faster.
Source: authors' analysis based on data from the VLSS 1992-1993 and SAIHS 1993

Let us focus on some of the more interesting cases, starting with Vietnam 1992-1993. As shown in Figure 2a, targeting group 2 should be prioritized in comparison with group 5 as this would allow a larger reduction in multidimensional poverty over most of the whole surface denoted by the poverty frontiers shown in that figure. The only relatively minor exceptions are the darkest areas. Move now to South Africa 1993. From Figure 2b we learn that group 15 should be preferred as compared to group 16. This is true for the entire area (captured bu ϕ^*) identified by the poverty frontiers used for this test. Given the dominance results provided in the theoretical section, these outputs found for Vietnam and South Africa can be extended to all of the bi-dimensional poverty indices belonging to the class $\Pi^{1,1}(\phi^*)$ of multidimensional poverty indices, and for all the poverty frontiers as denoted in that figure.

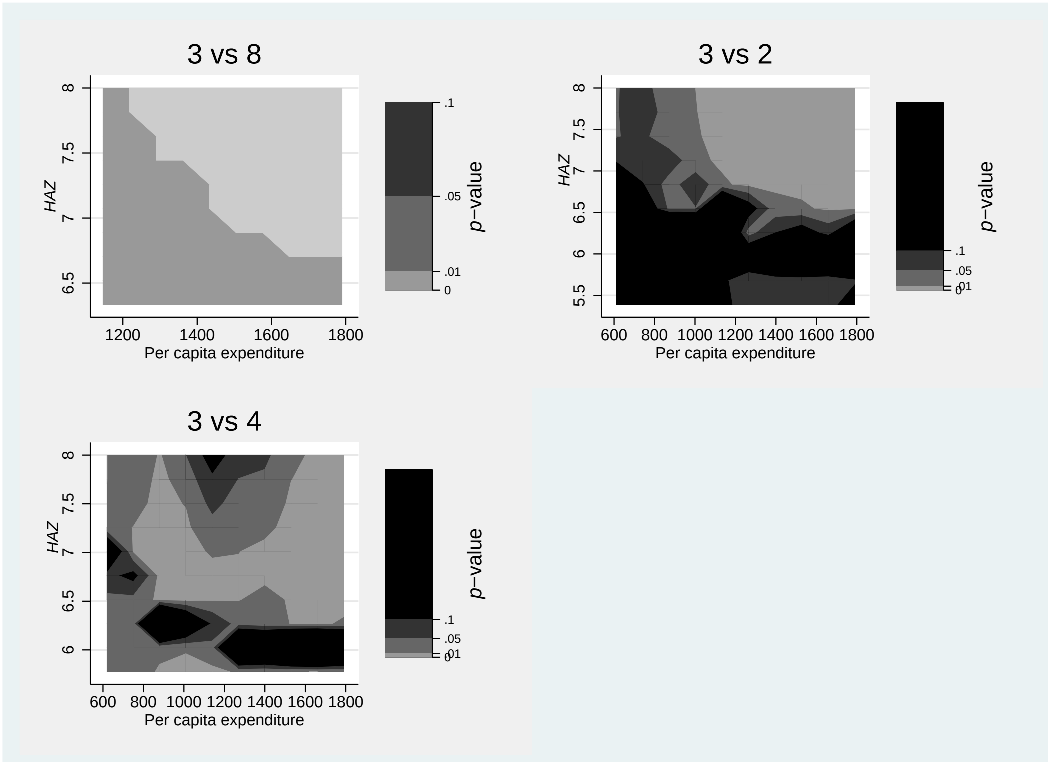
Let us now move to the class of bi-dimensional poverty indices $\Pi^{2,2}(\phi^*)$. As shown in Figure 3, targeting group 3 is preferable to targeting group 8 over the whole area of poverty frontiers shown in that figure. This says that reducing multidimensional poverty gap index in the dimensions of consumption and nutrition is faster when group 3 is targeted as opposed to group 8. Because of the dominance results stated in section 2.1, this also says that it is preferable to target group 3 for all of the bi-dimensional poverty indices that are members of the class $\Pi^{2,2}(\phi^*)$ of multidimensional poverty indices, for all of the poverty frontiers within the area shown in Figure 3. Relative to group 2, group 3 is dominant only for upper nutritional and monetary poverty lines. Group 3 dominates group 4 for much of the region traced by the poverty lines.

Consider now Figure 4 for South Africa. As discussed above, group 3 should be the group to be targeted first for $\alpha_x = \alpha_y = 1$. Figure 4 shows that this is true for a wide area of poverty frontiers and for the $\Pi^{2,2}(\phi^*)$ class of poverty indices. Targeting group 3 dominates targeting group 13 only over the region with monetary poverty lines above around 70 Rand and nutritional poverty lines above around 7.5. A more detailed examination of the results shows that while the conjunction of monetary and nutritional poverty (the first term on the right-hand side of (7)) does allow a robust ranking even for lower poverty lines, this is not anymore the case when the correlation component is added in. Targeting group 3 is preferable to targeting 16 in a univariate monetary perspective and over the the entire range of the poverty lines shown in the figure. At lower monetary and nutritional poverty lines, targeting group 3 is not anymore statistically preferable when poverty is expanded multidimensionally to consider also nutritional poverty.

As discussed above, reducing univariate or multidimensional poverty does not necessarily lead to the same policy agenda. It is theoretically clear, and empirically observable, that some socio-economic targeting scheme may be efficient at reducing univariate poverty, but may be sub-efficient at alleviating multidimensional poverty; the reverse is also true.

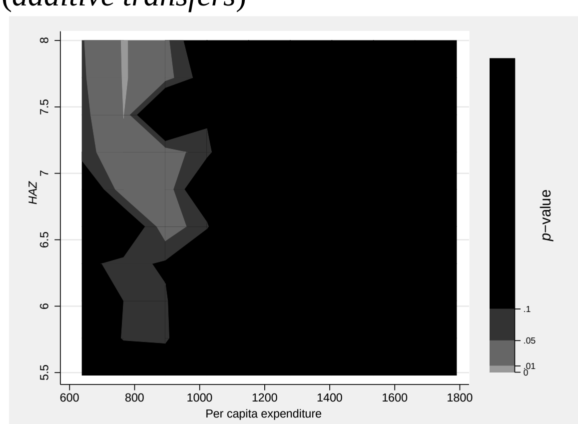
Consider the results shown in Figure 5 as an example of this. The Figure displays the poverty impact differences between targeting group 6 and targeting group 10, with respect to a relatively large area of poverty lines. From the perspective of alleviating unidimensional poverty in either dimension (Figure 5b), it is possible to favor group 6 over group 10 over relatively wide ranges of poverty lines (though results for nutritional dimension are robust only for poverty lines larger than 7.5). The targeting preference becomes, however, statistically insignificant when assessing multidimensional poverty over multidimensional combinations of these ranges (Figure 5a).

Figure 3: Testing targeting dominance of group 3 over other groups for the class of $\Pi^{2,2}$ indices (*additive transfers*)

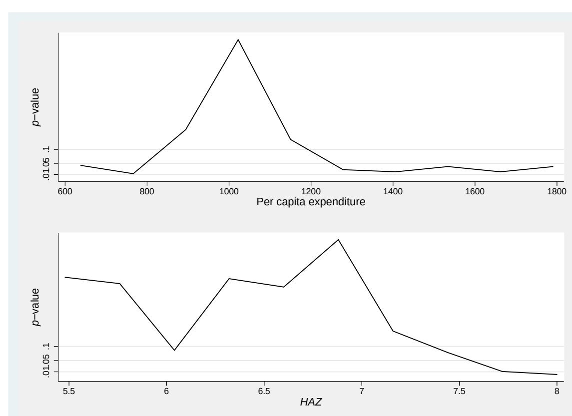


Note: the graphs show the p-values of the differences in poverty impact between targeting group 3 and targeting other groups; the lighter areas indicate where it is statistically more likely that targeting group 3 will reduce poverty faster.
Source: authors' analysis based on data from the VLSS 1992-1993

Figure 5: Testing targeting dominance of group 6 over group 10 for the class of $\Pi^{2,2}$ indices (*additive transfers*)



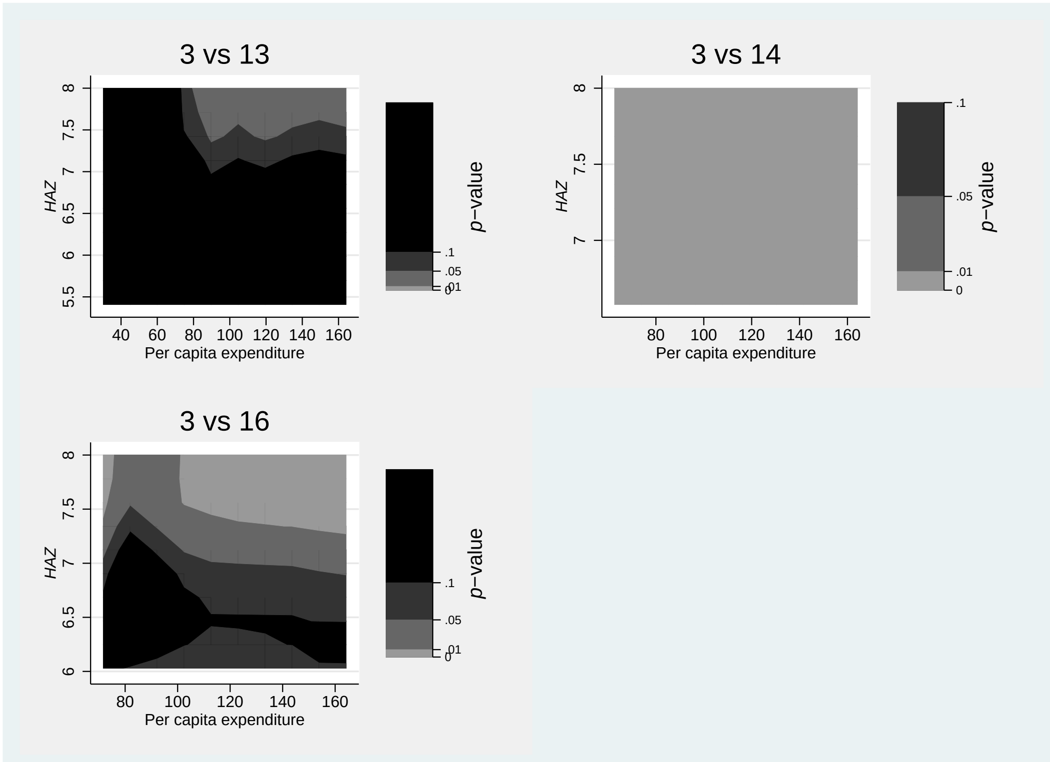
(a) multidimensional



(b) unidimensional

Note: the first graph shows the p-values of the differences in poverty impact between targeting group 6 and targeting group 10; the lighter areas indicate where it is statistically more likely that targeting group 6 will reduce poverty faster. The second graph shows the p-values of the difference in the unidimensional poverty impact between targeting group 6 and targeting group 10.
Source: authors' analysis based on data from the VLSS 1992-1993

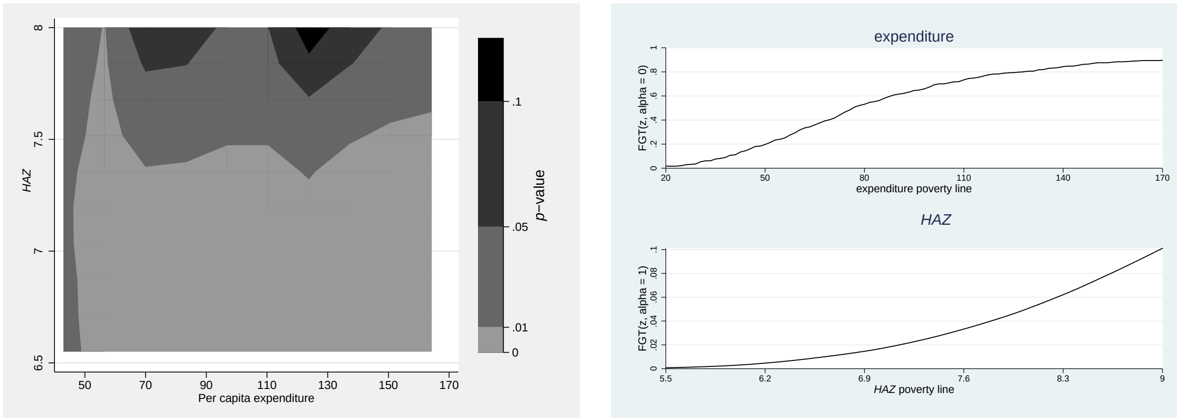
Figure 4: Testing targeting dominance of group 3 over other groups for the class of $\Pi^{2,2}$ indices (*additive transfers*)



Note: the graphs show the p-values of the differences in poverty impact between targeting group 3 and targeting other groups; the lighter areas indicate where it is statistically more likely that targeting group 3 will reduce poverty faster.
Source: authors' analysis based on data from the SAIHS 1993.

The converse result is observed in 1993 South Africa. This is shown in Figure 6 by assessing dominance of targeting group 9 over group 13. Although for most poverty lines neither univariate targeting dominance is observed, for a large area of multidimensional combinations of these unidimensional poverty lines, the reduction in bi-dimensional poverty through targeting group 9 dominates statistically the reduction from targeting group 13.

Figure 6: Testing the dominance of targeting group 9 over group 13 (*additive transfers*)



(a) multidimensional (b) unidimensional

Note: the first graph shows the p-values of the differences in poverty impact between targeting group 9 and targeting group 13; the lighter areas indicate where it is statistically more likely that targeting group 9 will reduce poverty faster. The second graph shows the p-values of the difference in the unidimensional poverty impact between targeting group 9 and targeting group 13.
Source: authors' analysis based on data from the SAIHS 1993.

4 Conclusion

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